# Learning Data-Driven Uncertainty sets with Mean Robust Optimization

Bartolomeo Stellato

PRINCETON UNIVERSITY

ICSP 2025

# Joint work with

**Irina Wang**
Princeton ORFE

**Marta Fochesato**
ETH Zürich

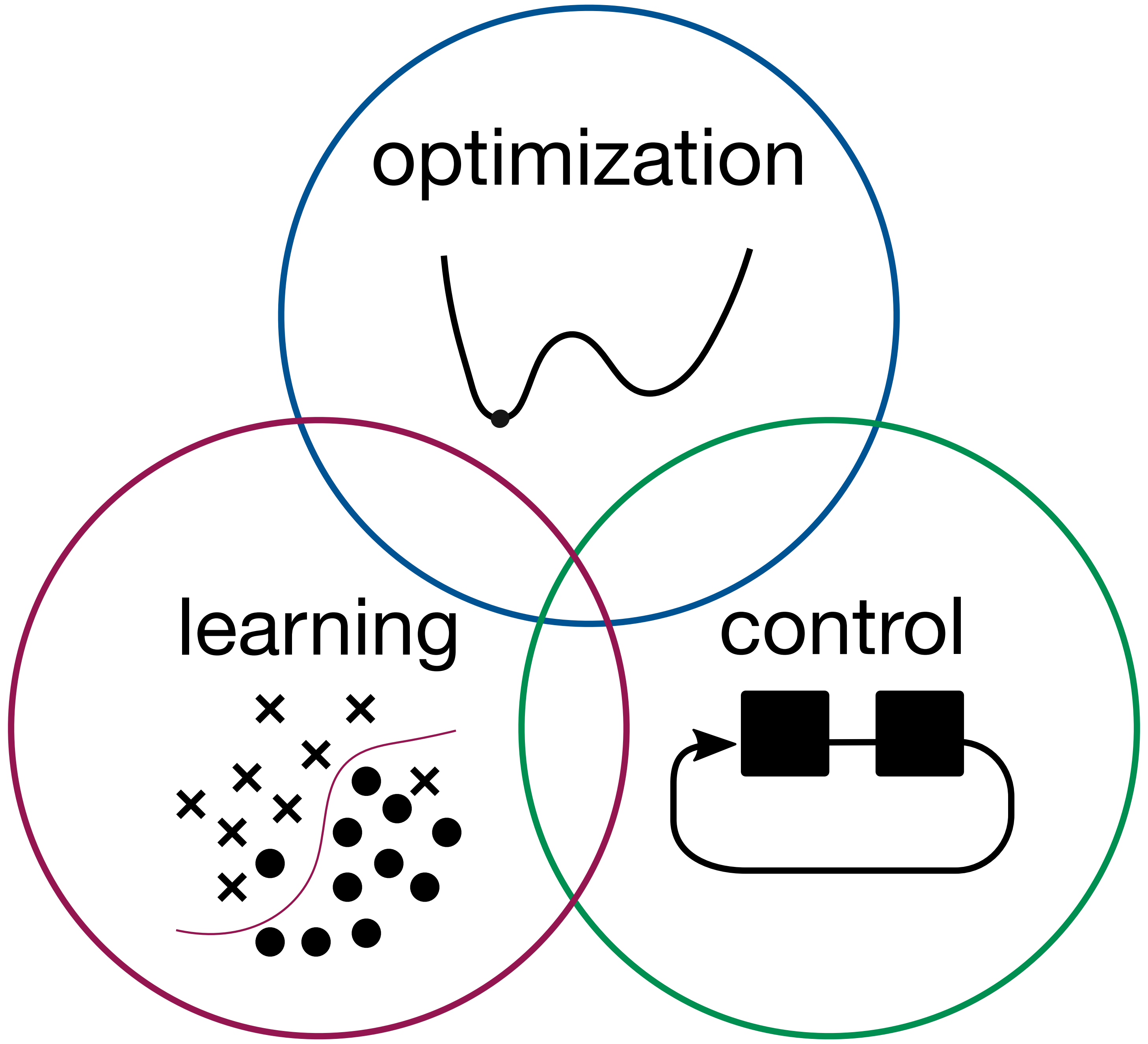
**Bart Van Parys**
CWI - the Netherlands

Marta Fochesato acknowledges the generous support of G-Research

# Most applications require fast and safe decisions in presence of uncertainty

# My research in a 🥜shell

**theory**



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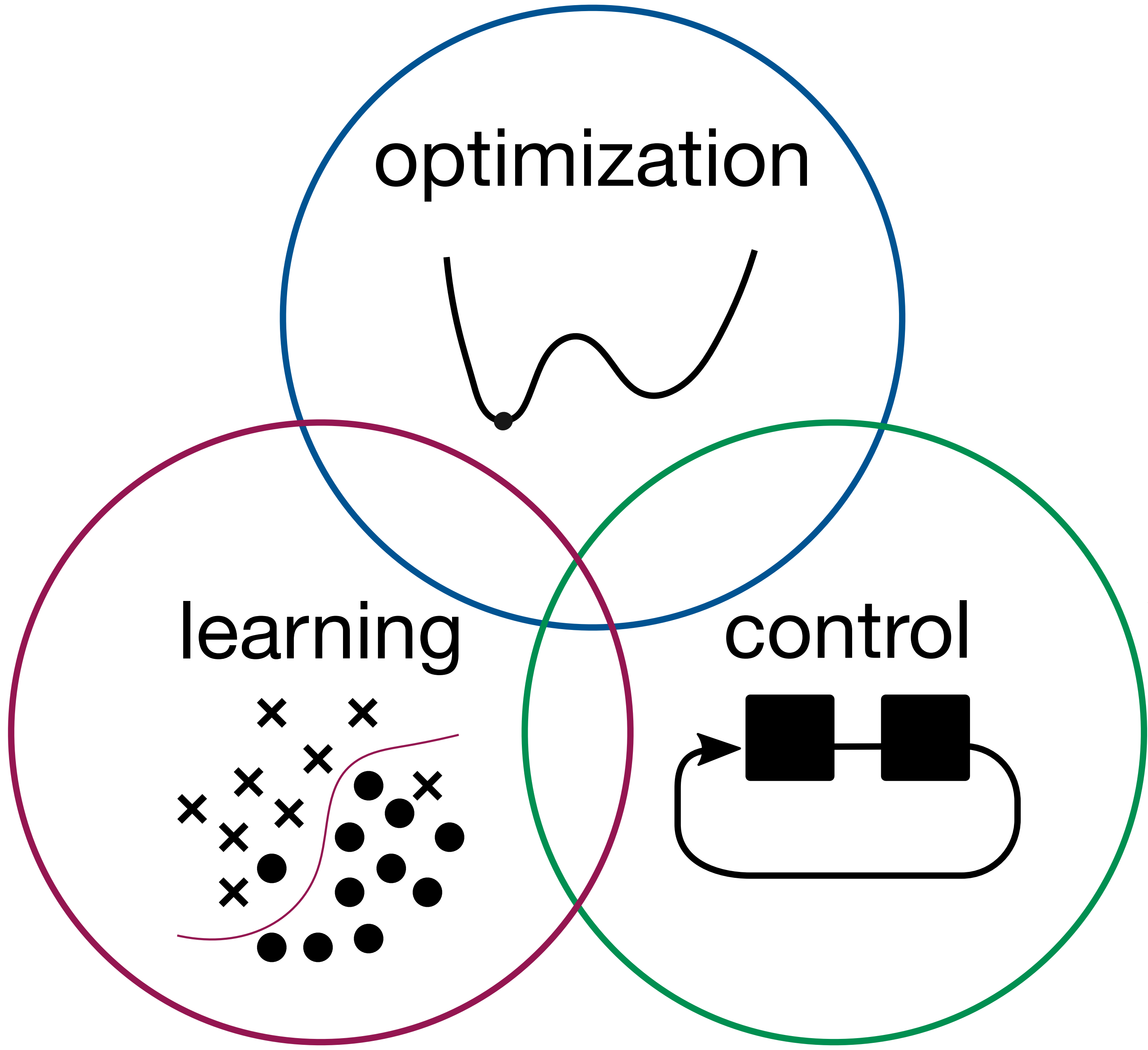

optimization

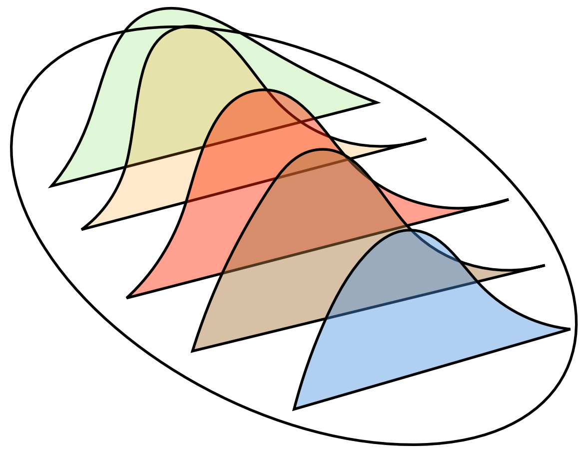
learning

control

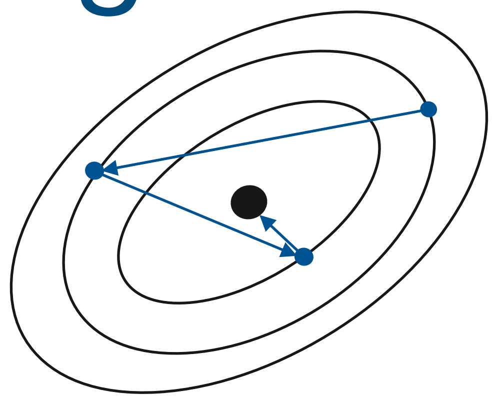
↔

## methodology

optimization under uncertainty



real-time optimization algorithms



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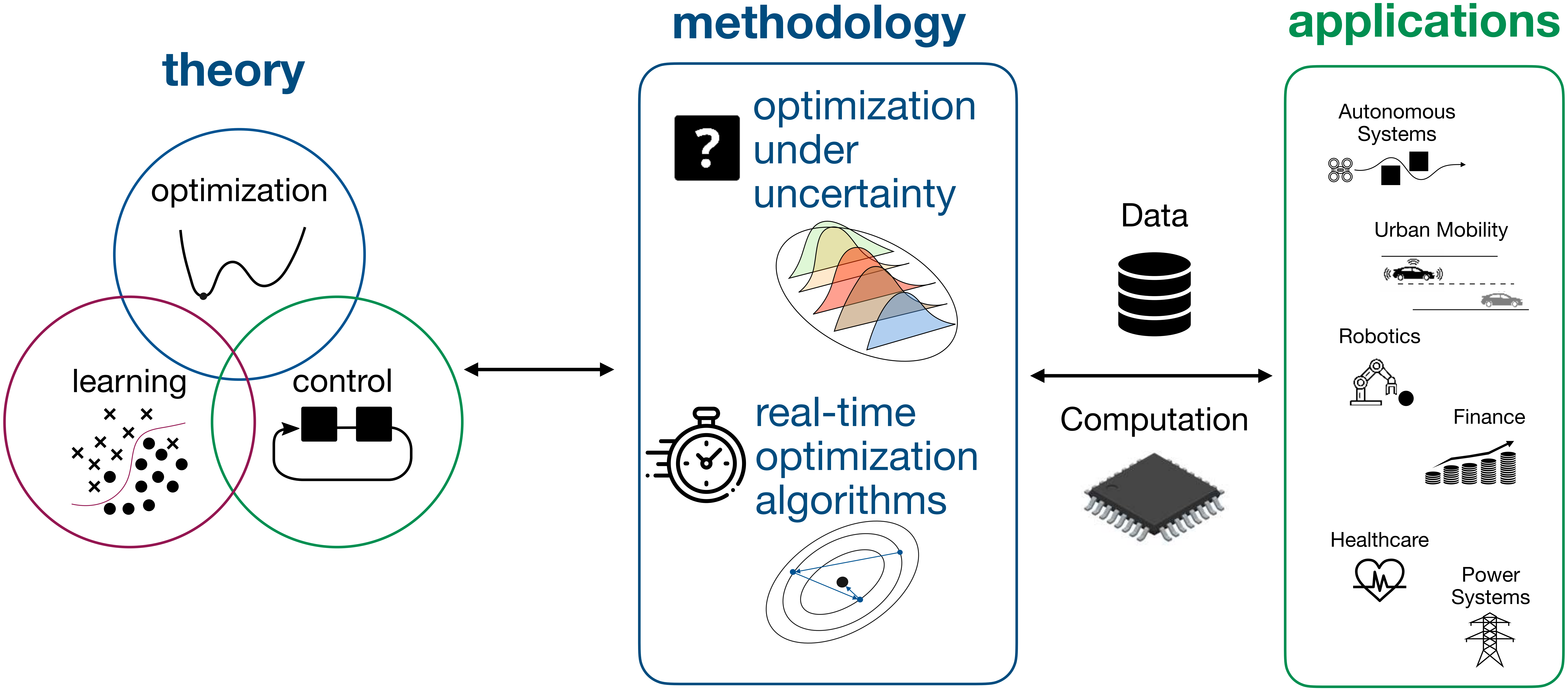

## theory

optimization

learning

control

## methodology

optimization under uncertainty

real-time optimization algorithms

Data

Computation

## applications

Autonomous Systems

Urban Mobility

Robotics

Finance

Healthcare

Power Systems

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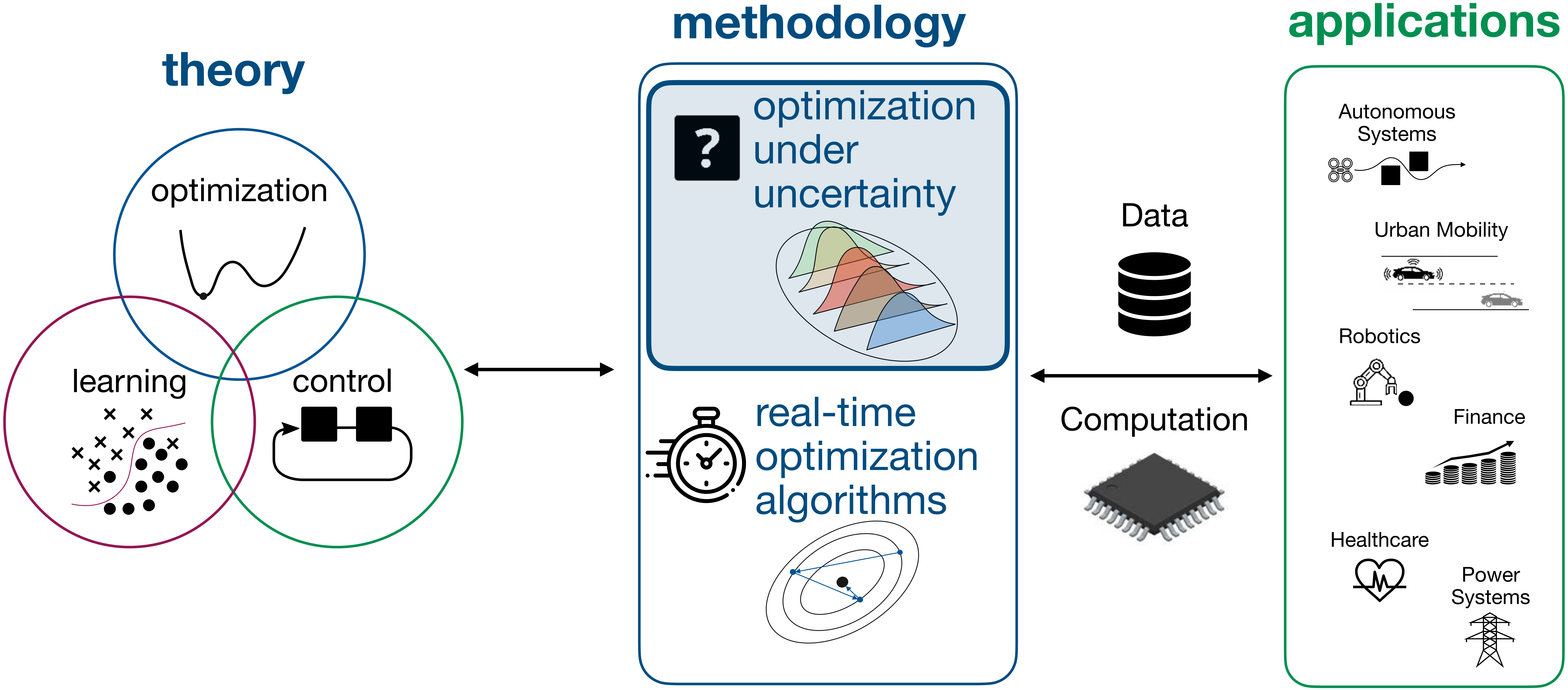

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# Data can help us but we have to use it wisely!

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Moore's Law is slowing down

The New York Times

*Moore's Law Running Out of Room, Tech Looks for a Successor*

The Economist explains

The end of Moore's law

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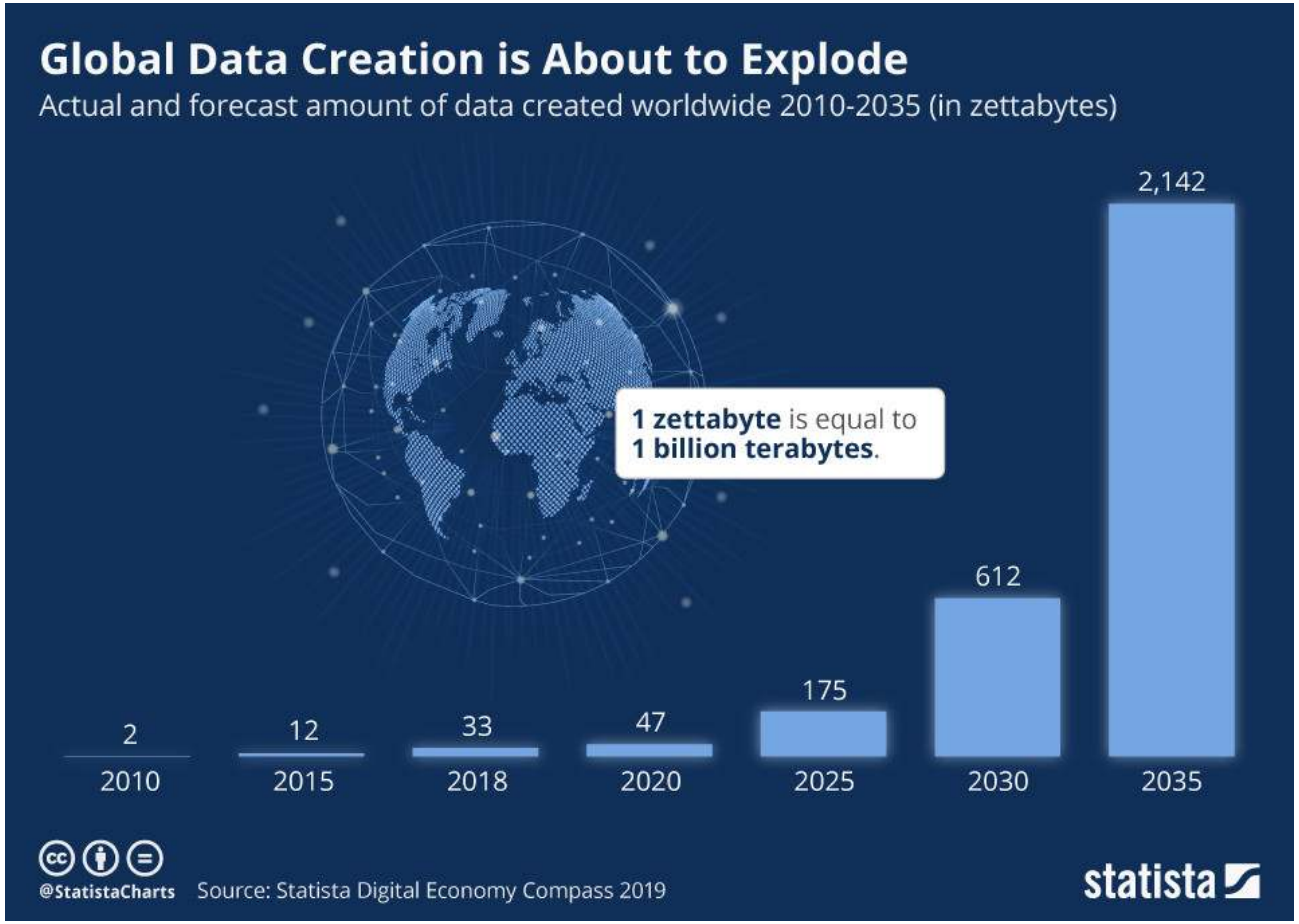
## Data availability is exploding



**Global Data Creation is About to Explode**

Actual and forecast amount of data created worldwide 2010-2035 (in zettabytes)

Source: Statista Digital Economy Compass 2019

# Problem setup with uncertain constraints

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g(x,u) \leq 0 \end{array}$$

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variable

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How do we guarantee constraint satisfaction?

# Finite sample probabilistic guarantees in expectation

$$\mathbf{E}(g(x, u)) \leq 0$$

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$u \sim P$

(but we never know $P$!)

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Data-driven probabilistic guarantees

$$\mathbf{P}^N \left(\mathbf{E}(g(\hat{x}_N, u)) \leq 0\right) \geq 1 - \beta$$

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product distribution →

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data-driven solution

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product distribution

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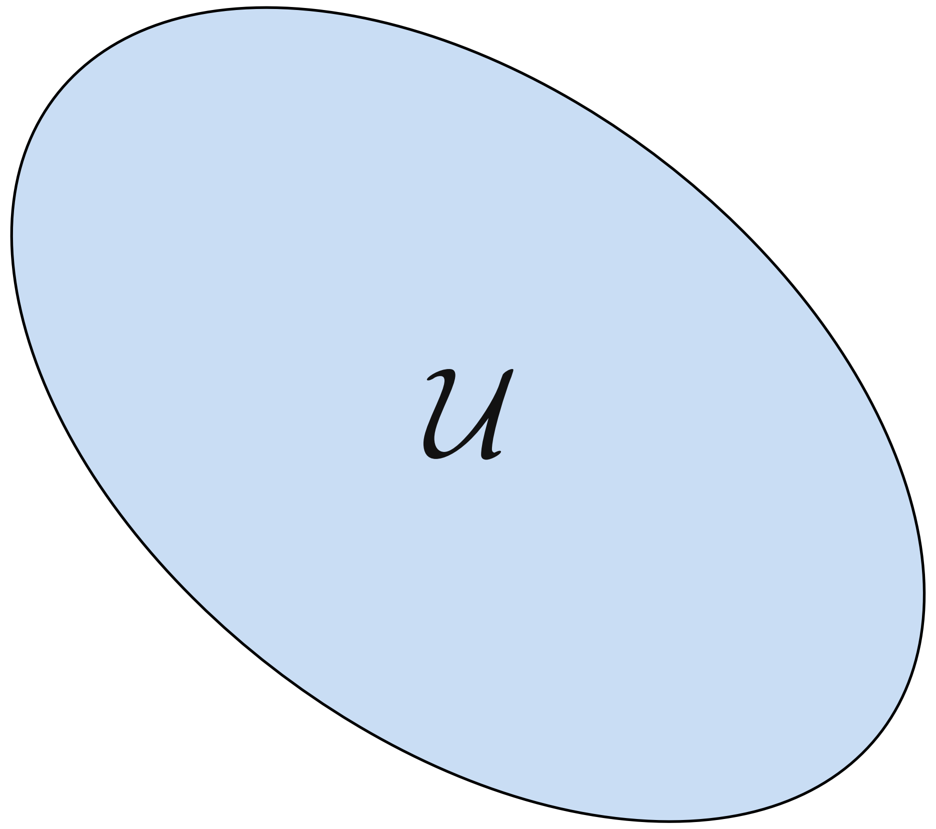
probability of constraint satisfaction

# Robust optimization recipe

### Recipe

1. Pick uncertainty set $\mathcal{U}$
2. Ensure constraint satisfaction $\forall u \in \mathcal{U}$



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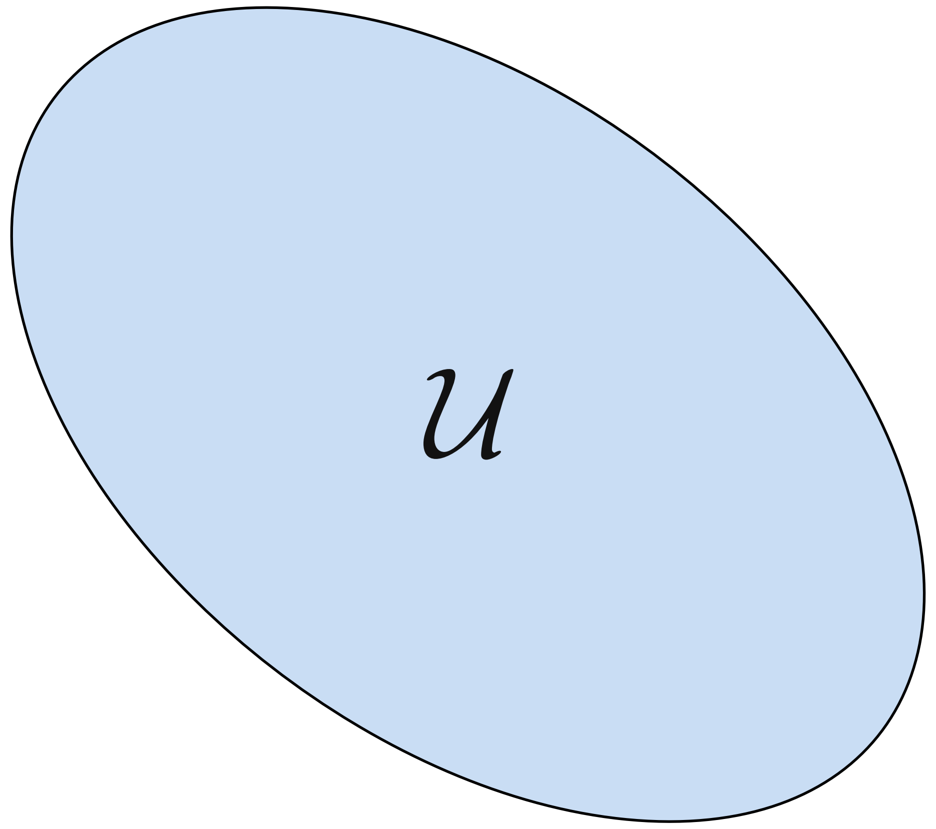

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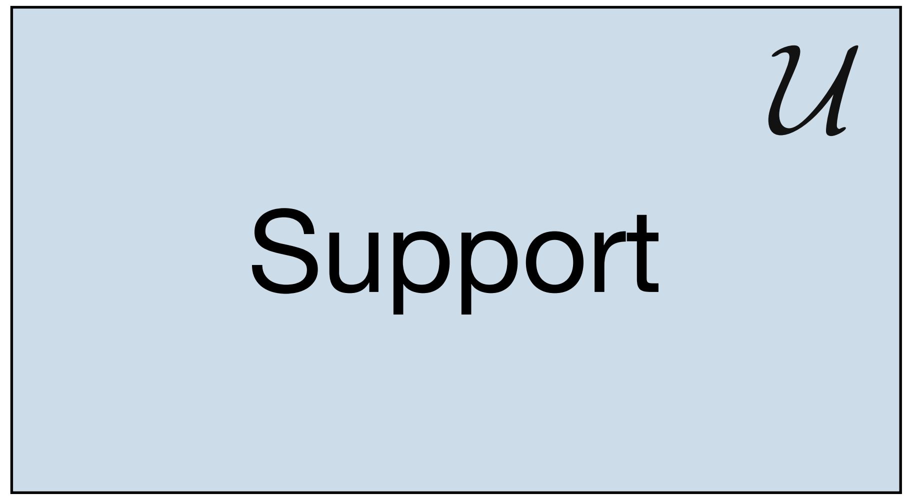
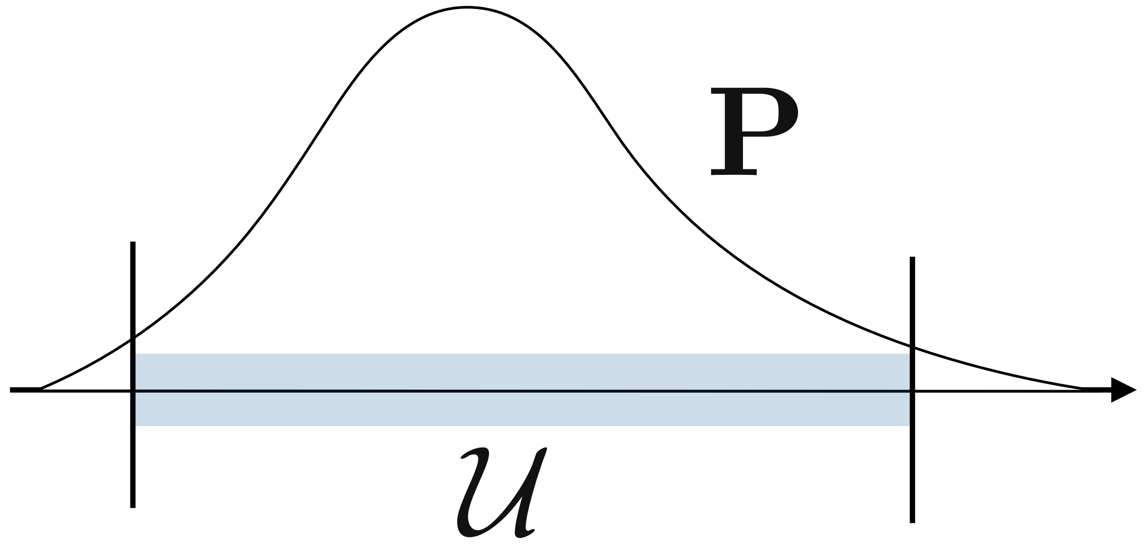
How do we pick the uncertainty set?

# Constructing the uncertainty set is difficult

*data-free approaches*



Worst-case approach



Probabilistic approach

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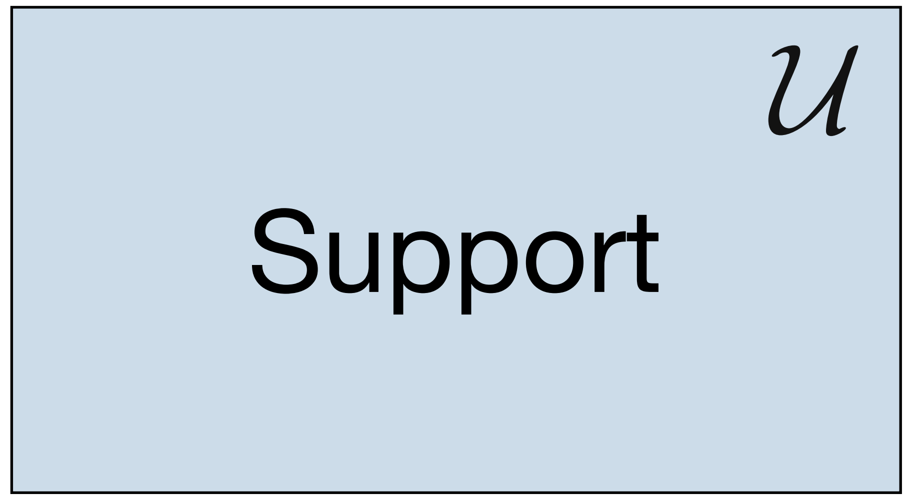

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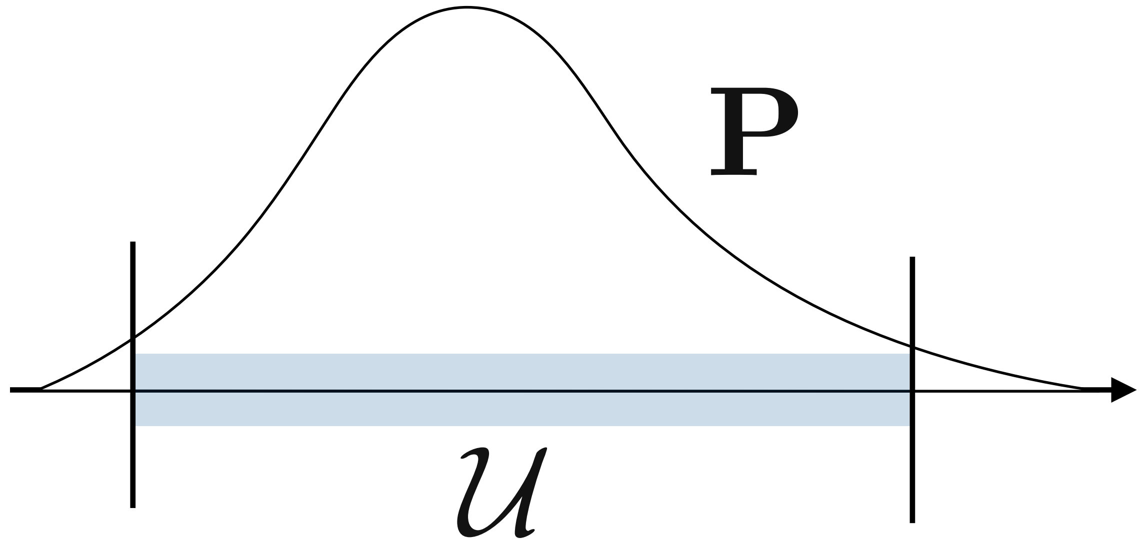

Probabilistic approach

**rely on *a-priori* pessimistic assumptions**

El Ghaoui, Ben-Tal, Bertsimas, Nemirovski, Sim, den Hertog…

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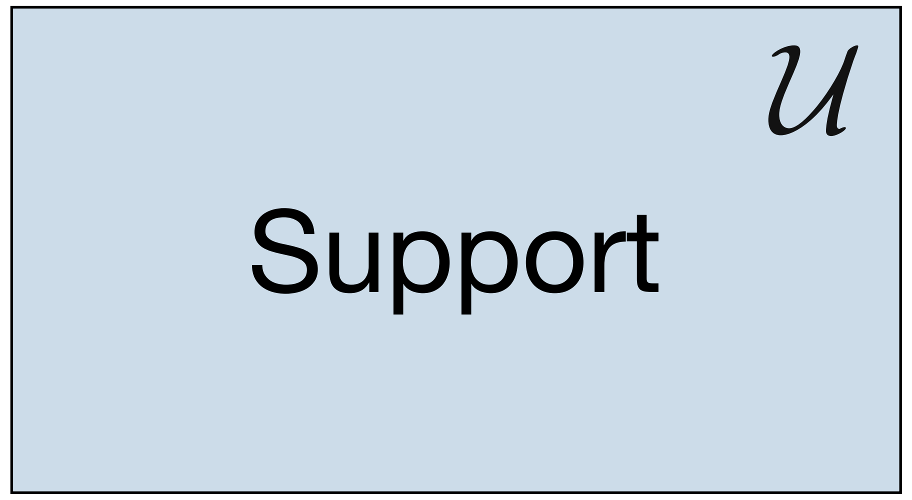

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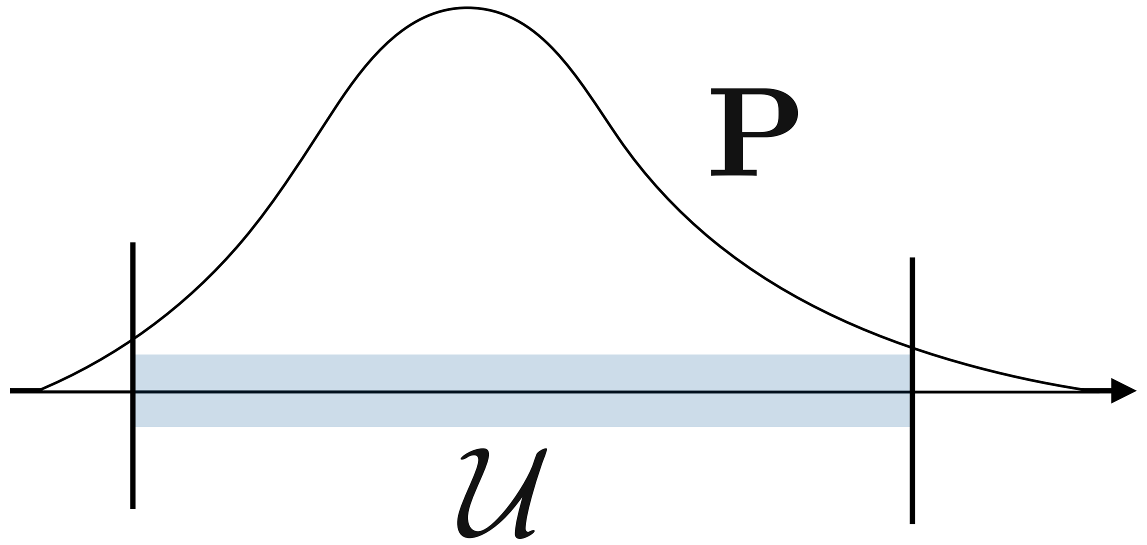

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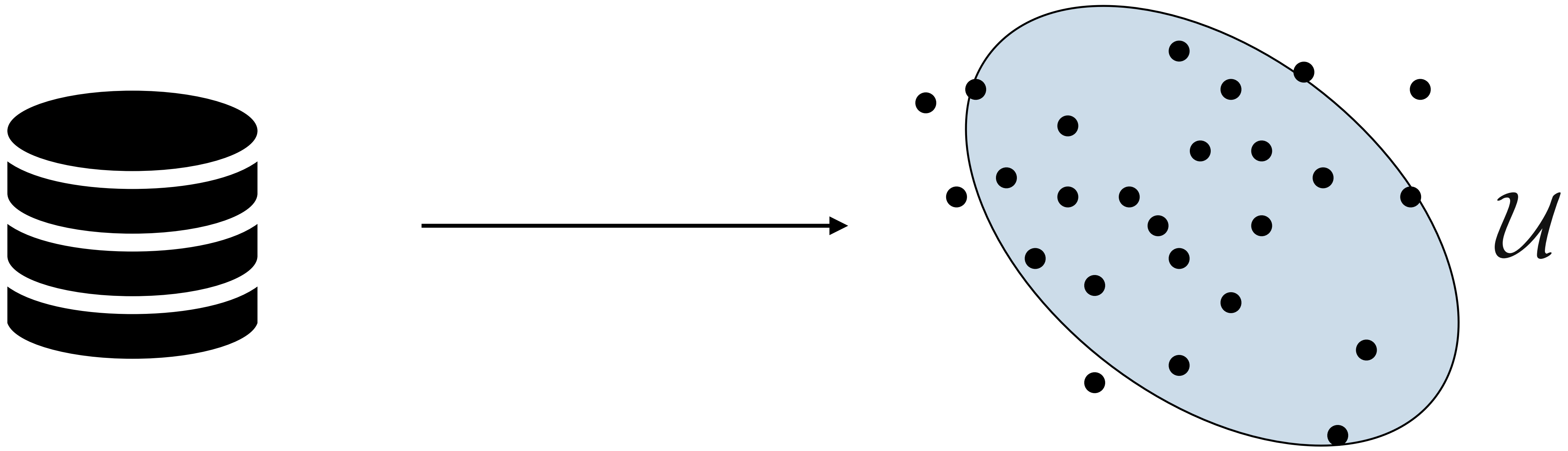

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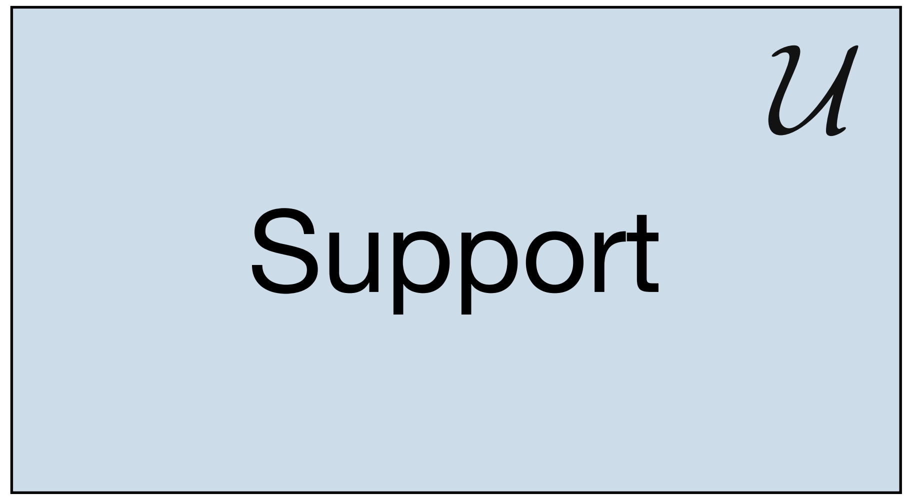

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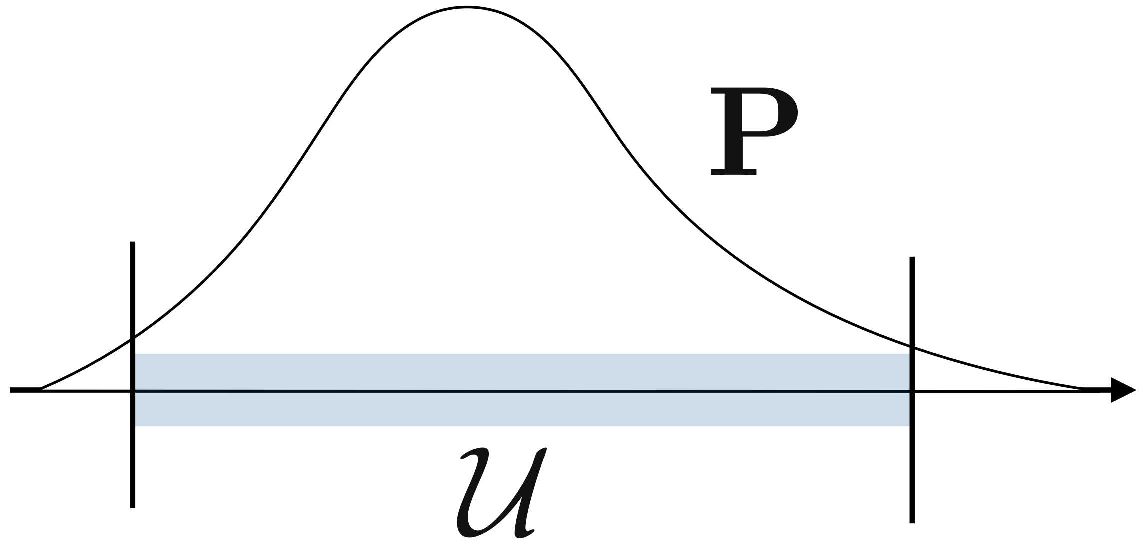

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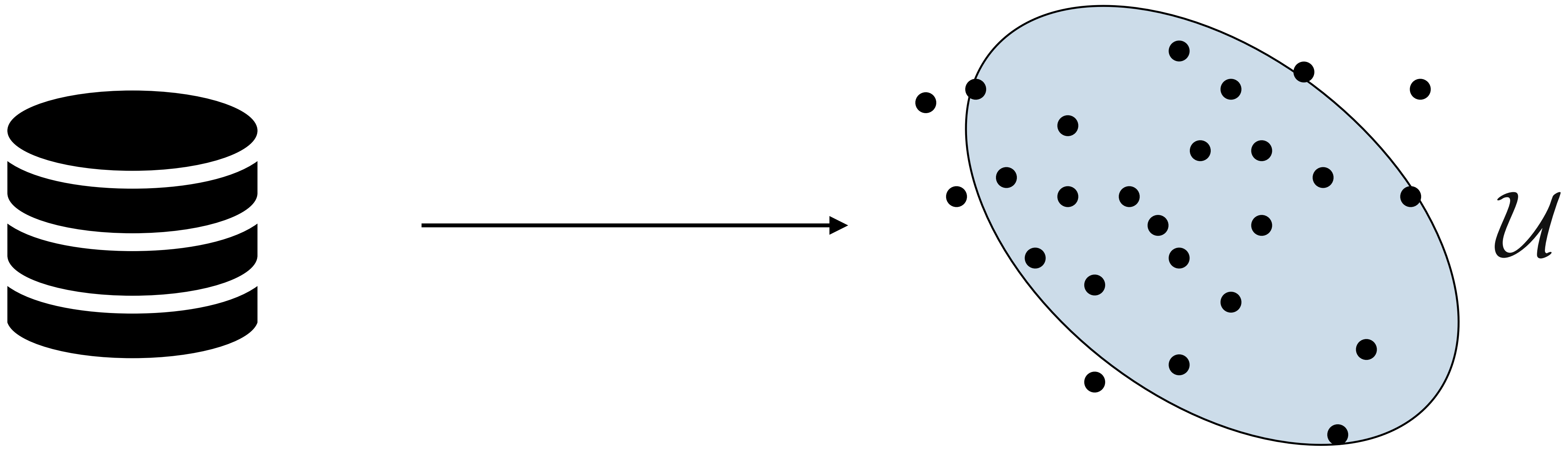

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Bertsimas, Gupta, Kallus (2014)

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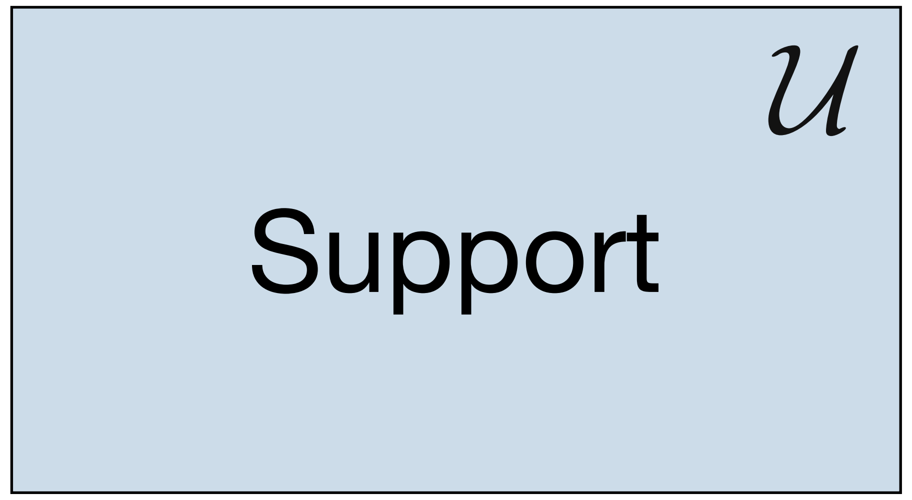

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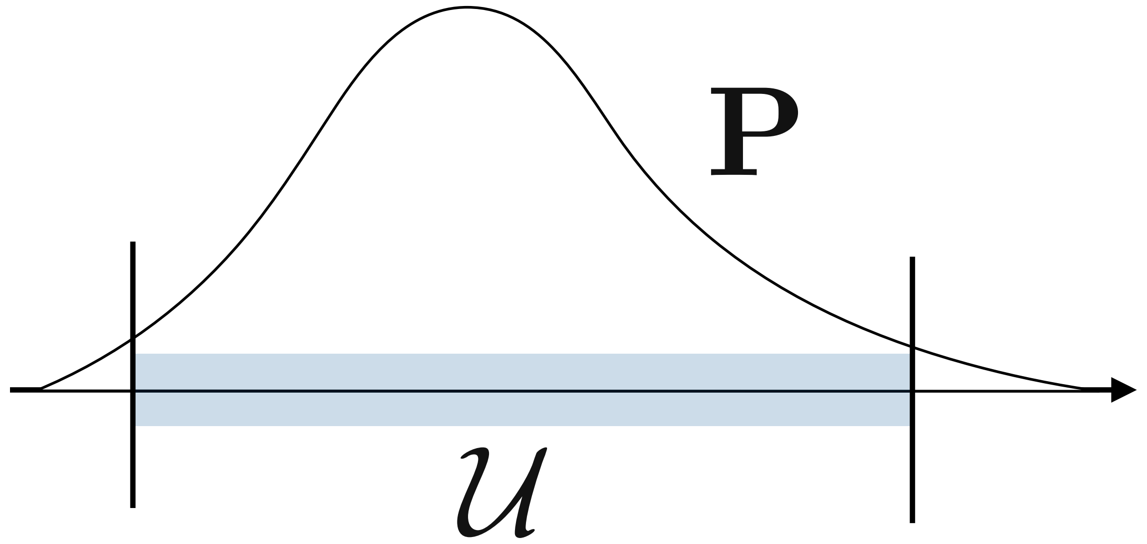

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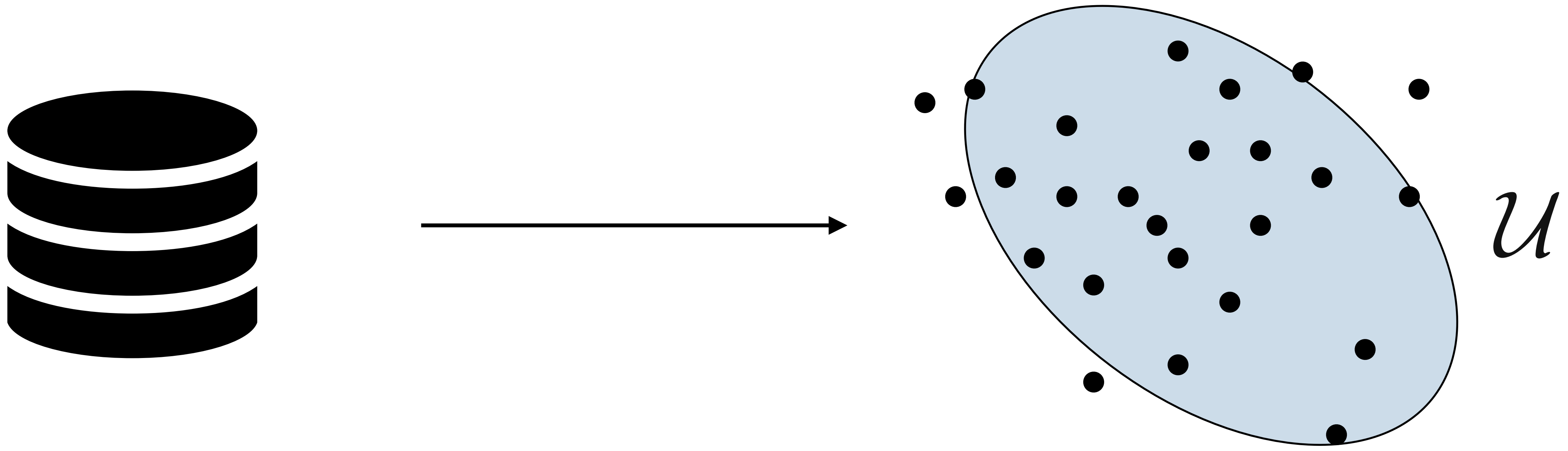

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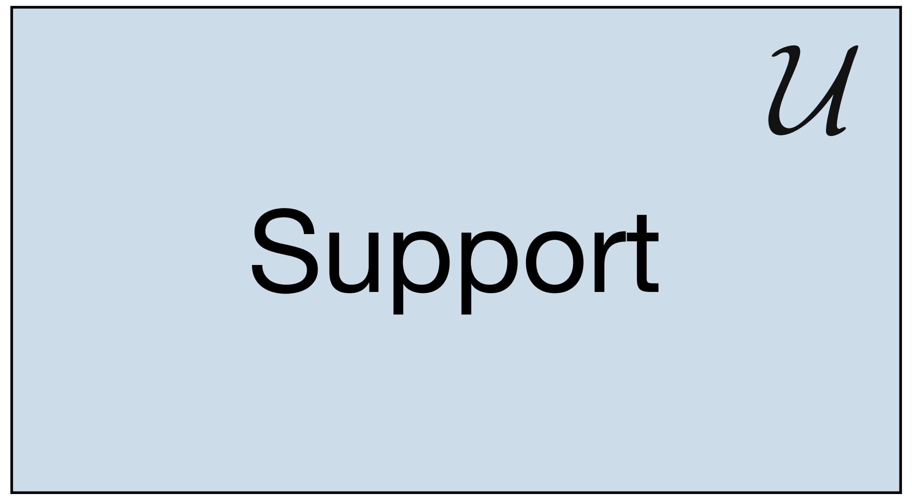

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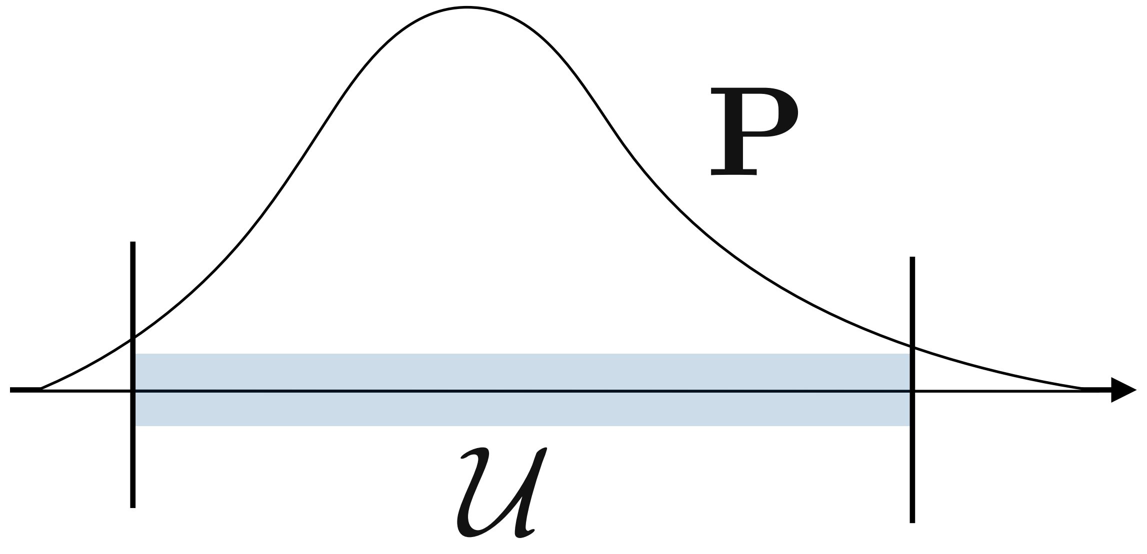

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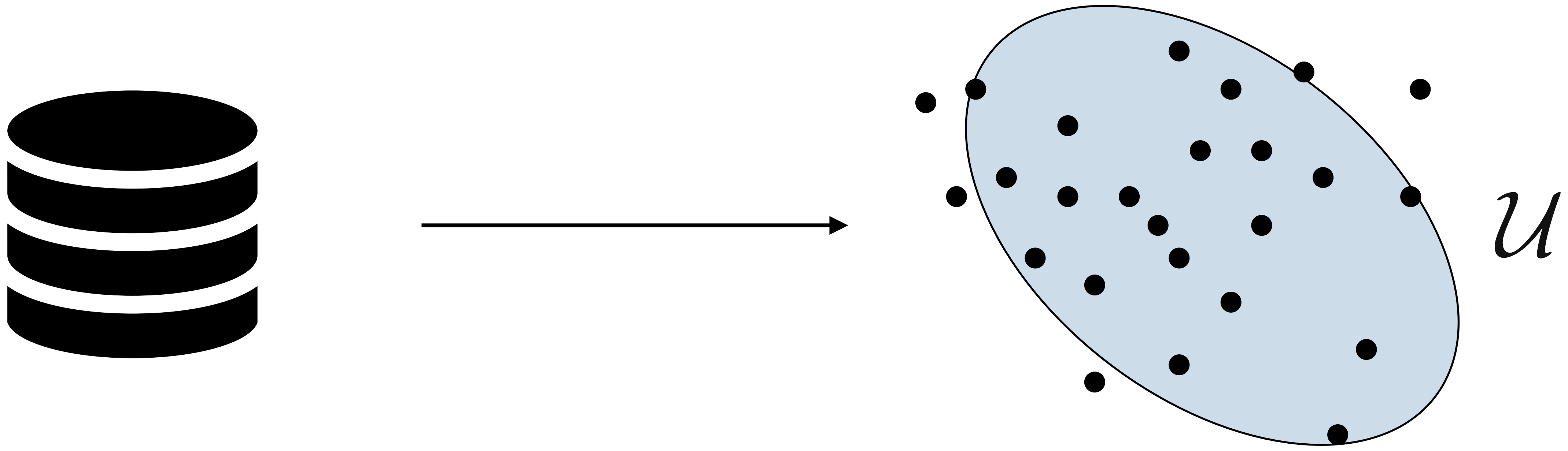

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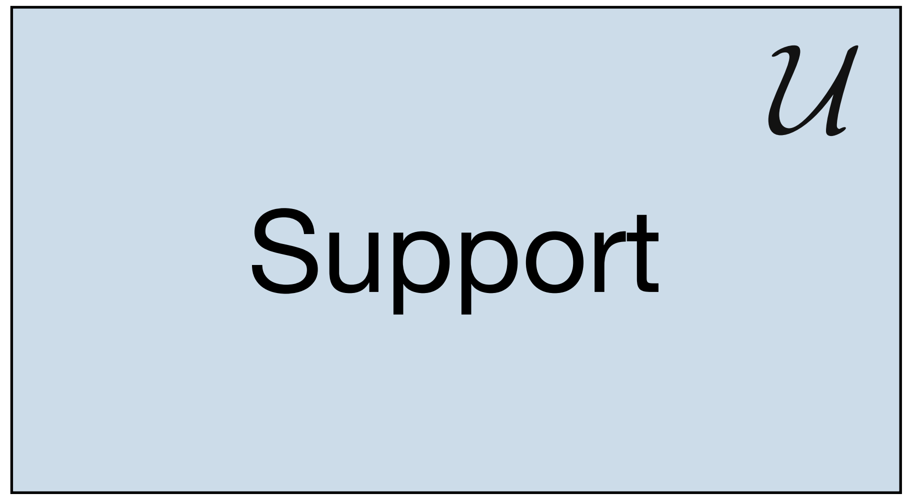

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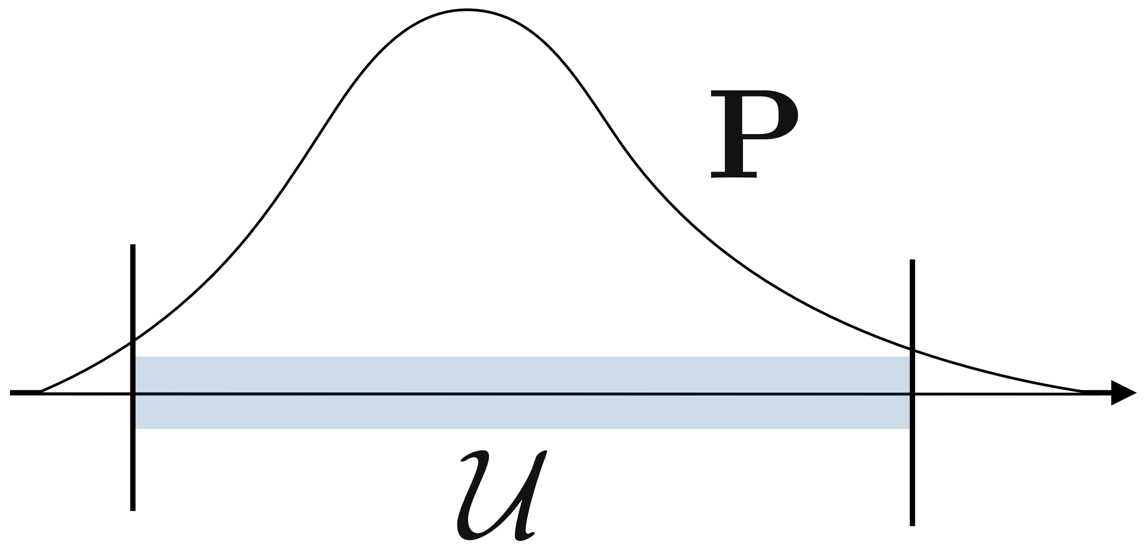

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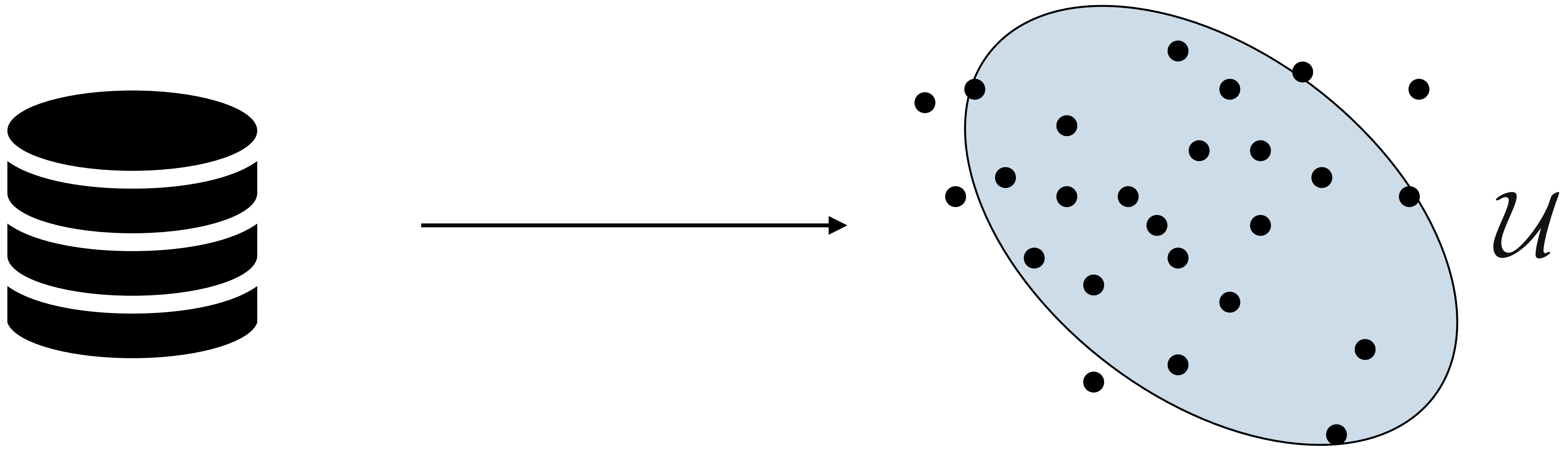

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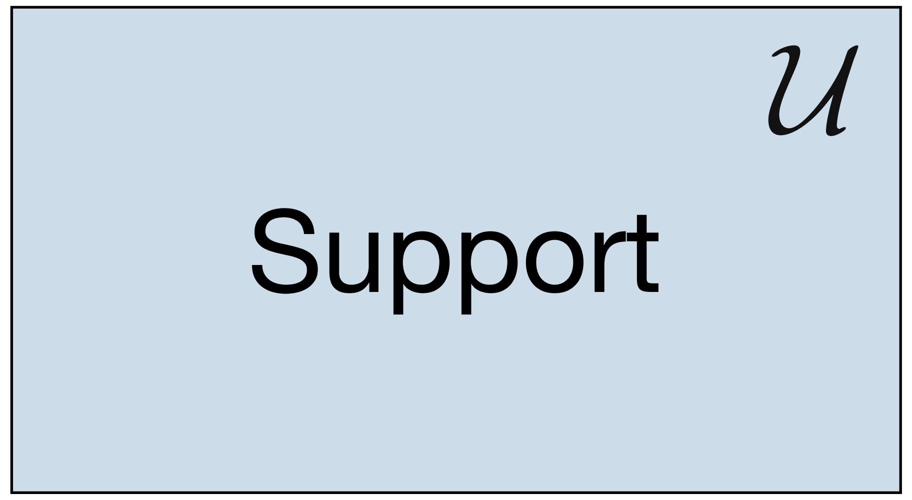

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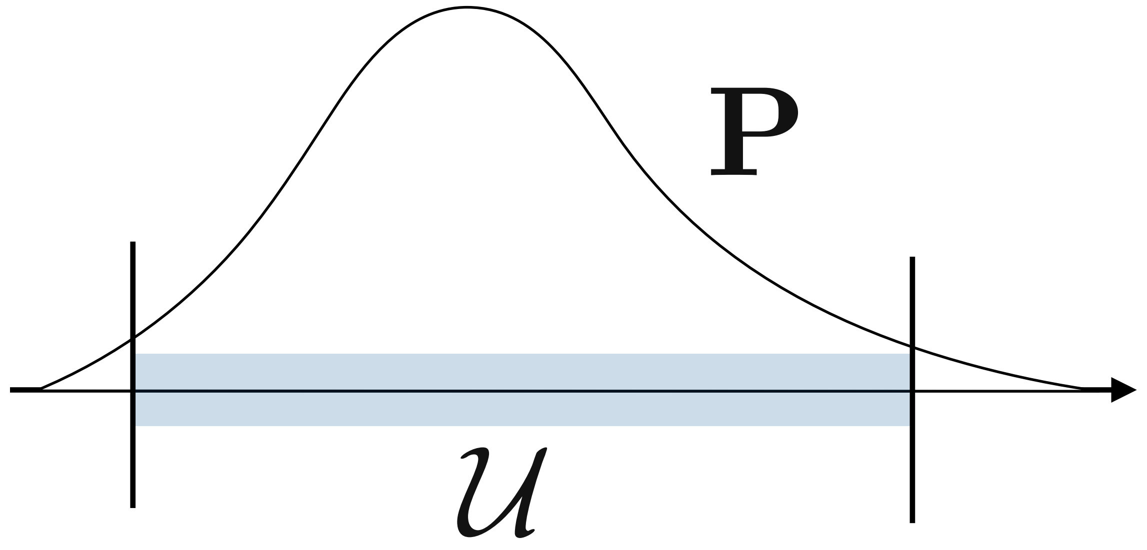

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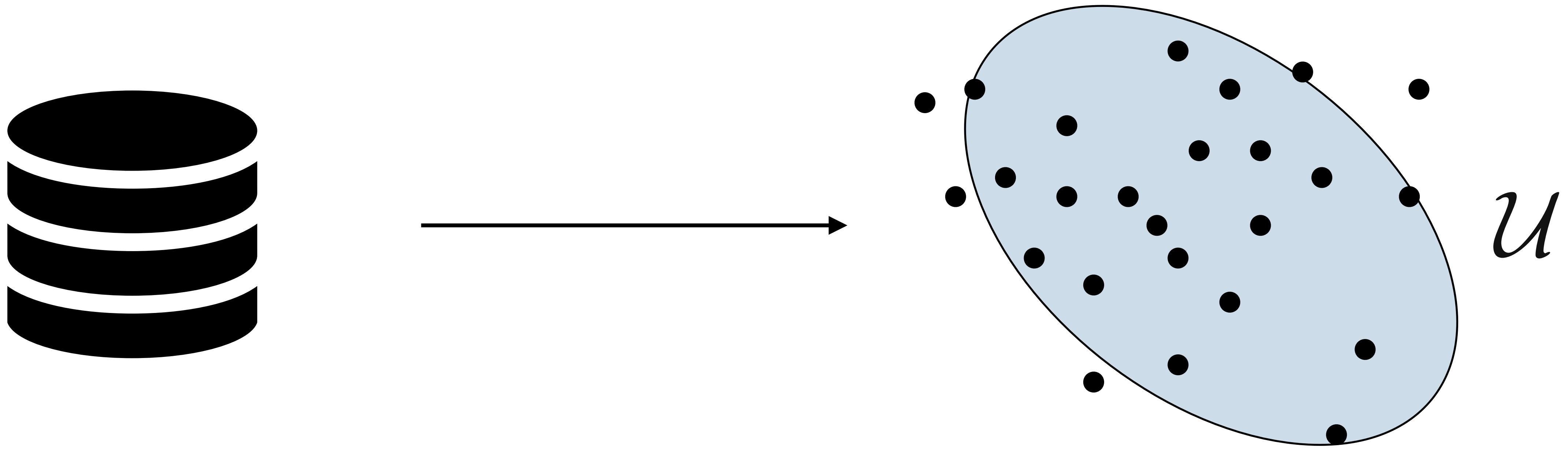

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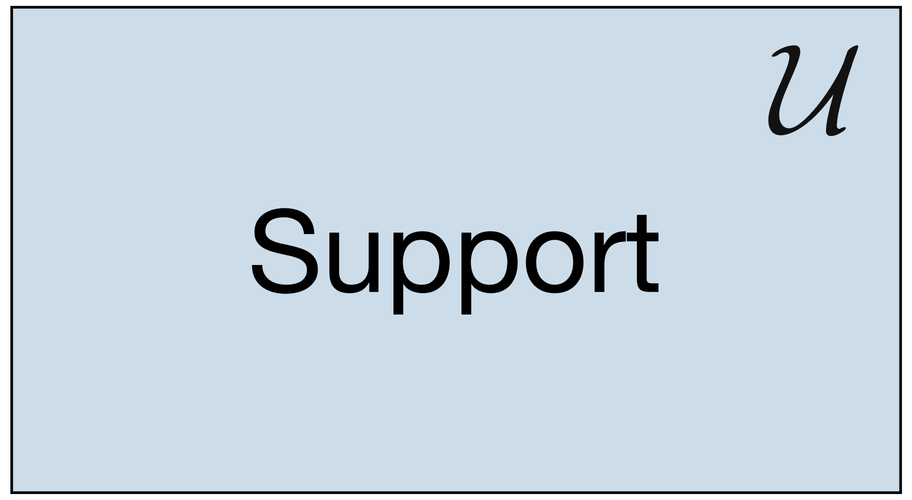

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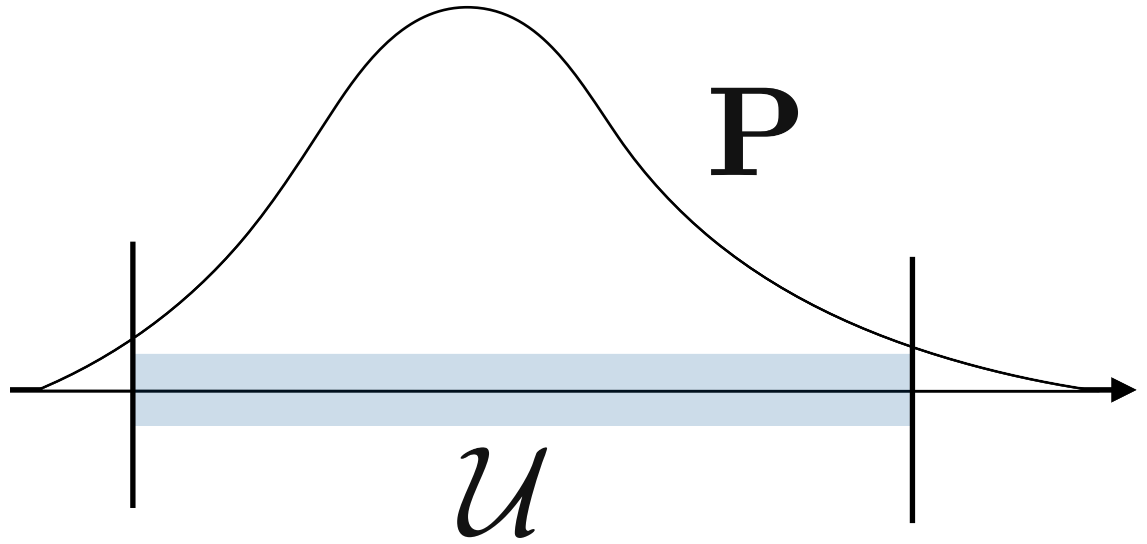

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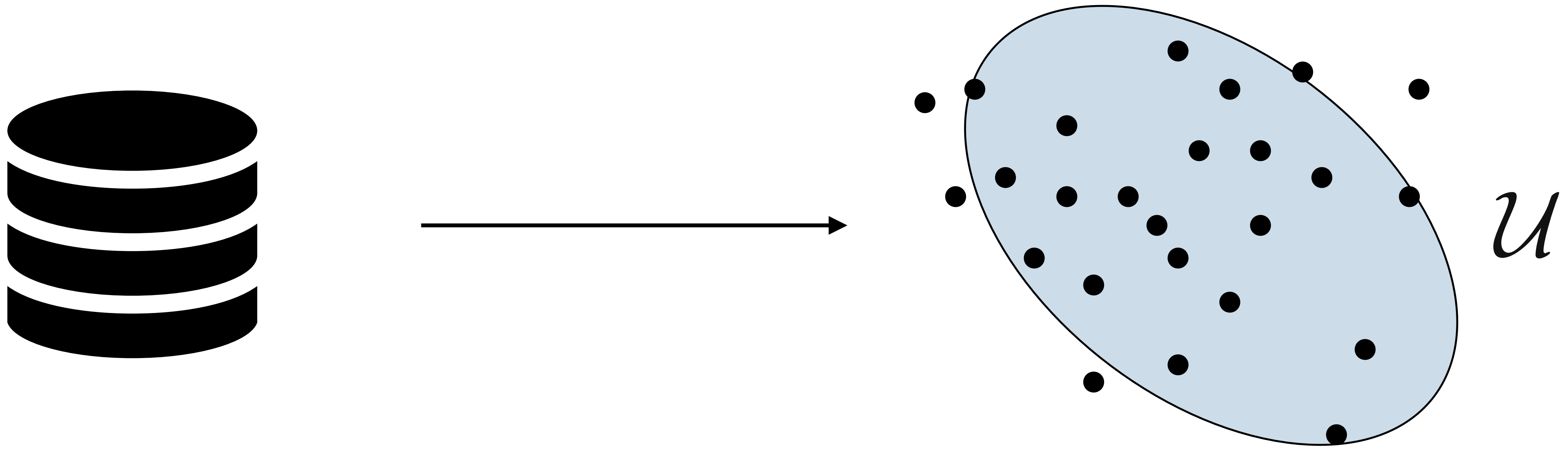

**Irina Wang's Talk**
Jul 30, 2025, 12:20 PM
Caquot

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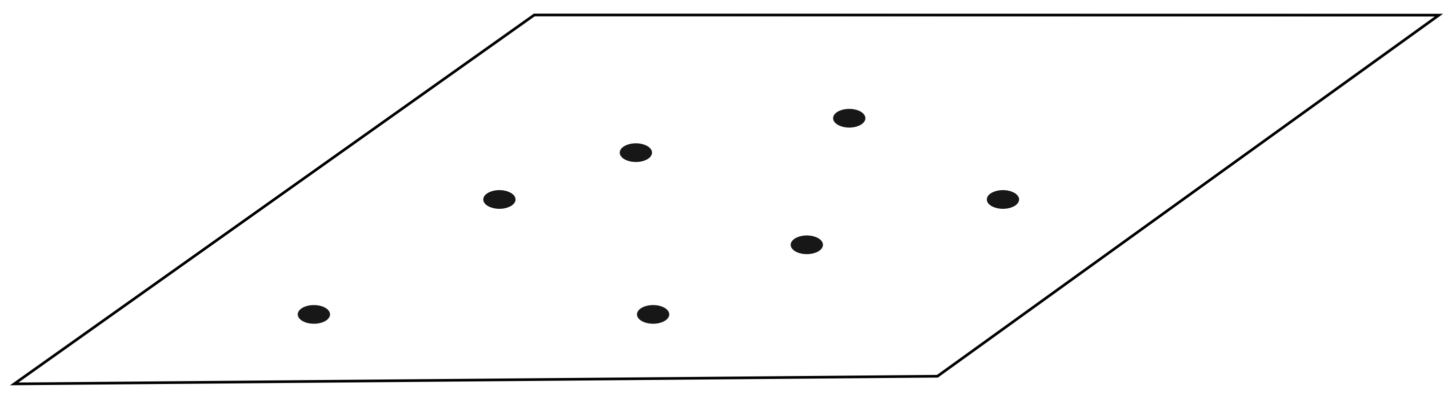
# The empirical distribution can help us build uncertainty sets

## Data-Driven Distributionally Robust Optimization

Data

$$\mathcal{D} = \{d_i\}_{i=1}^{N}$$



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## Data-Driven Distributionally Robust Optimization

### Empirical Distribution

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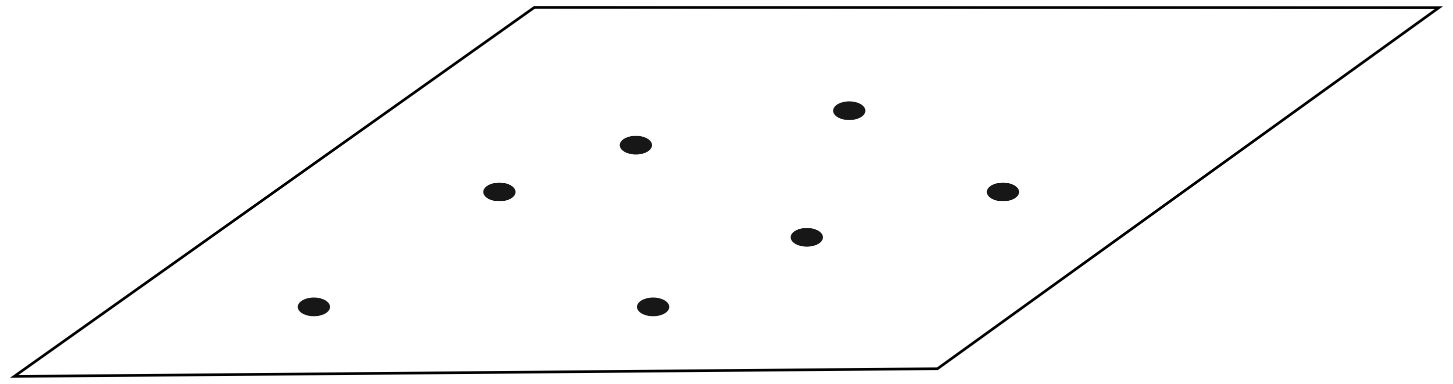
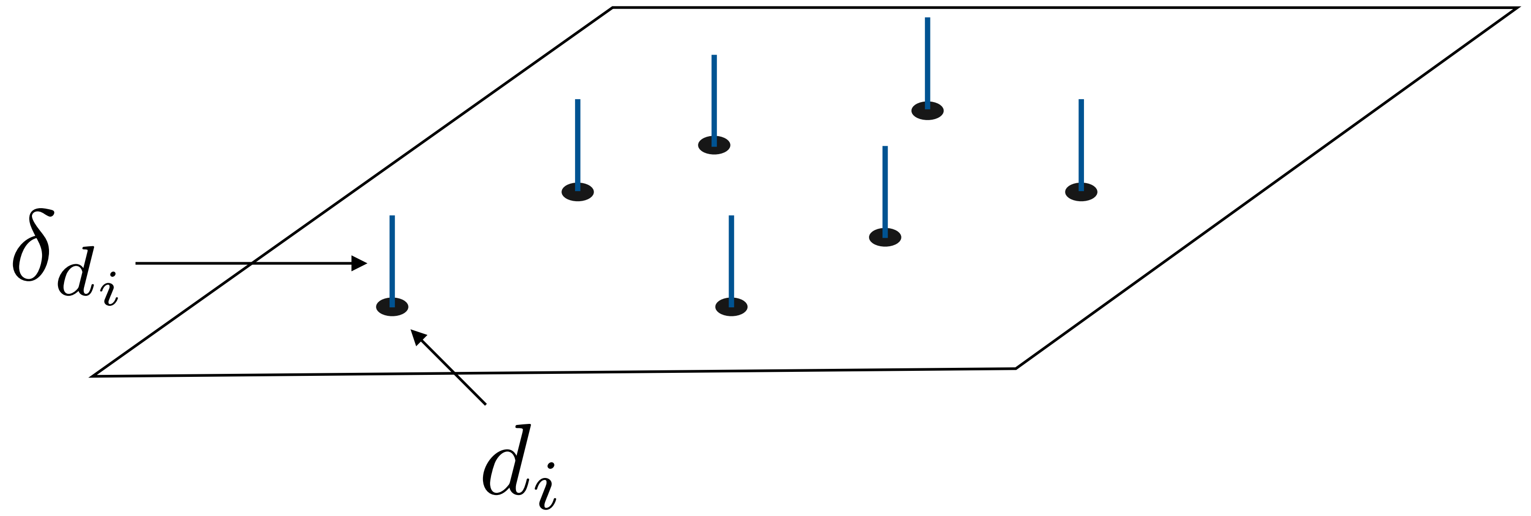
$$\mathcal{D} = \{d_i\}_{i=1}^{N} \longrightarrow \hat{\mathbf{P}}^N = \frac{1}{N}\sum_{i=1}^{N} \delta_{d_i}$$





# The empirical distribution can help us build uncertainty sets

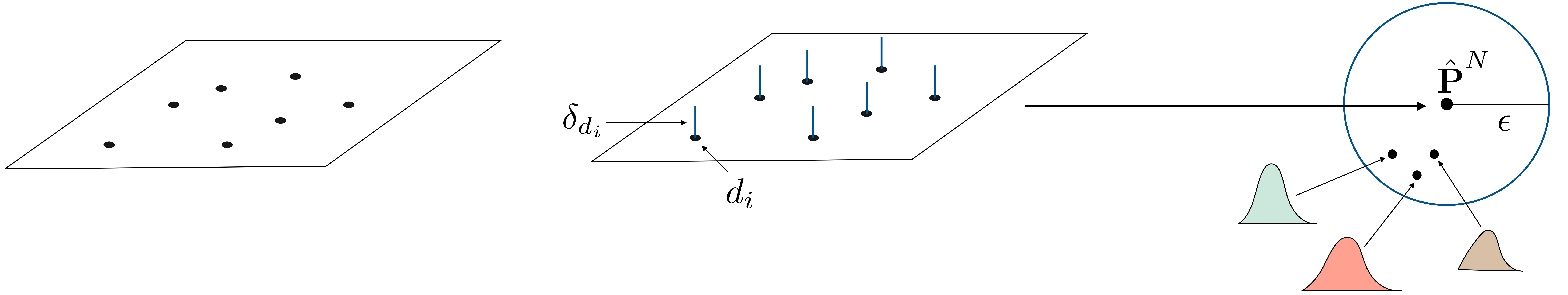
## Data-Driven Distributionally Robust Optimization

Data

**Empirical Distribution**

**Ambiguity Set**

$$\mathcal{D} = \{d_i\}_{i=1}^{N} \longrightarrow \hat{\mathbf{P}}^{N} = \frac{1}{N}\sum_{i=1}^{N} \delta_{d_i} \longrightarrow \mathcal{P}_N = \left\{ W_p(\hat{\mathbf{P}}^{N}, \mathbf{P}) \leq \epsilon \right\}$$



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## Data-Driven Distributionally Robust Optimization

### Data

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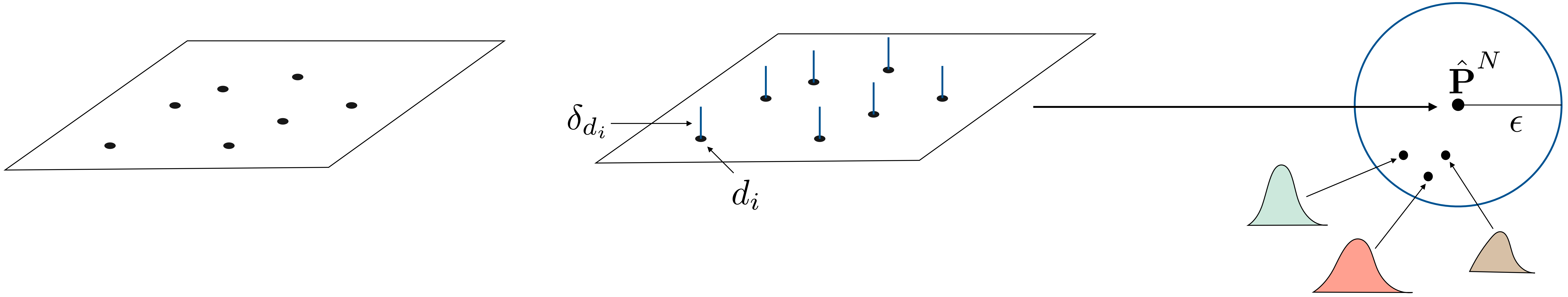
### Empirical Distribution

$$\hat{\mathbf{P}}^{N} = \frac{1}{N}\sum_{i=1}^{N}\delta_{d_i} \longrightarrow$$

### Ambiguity Set

$$\mathcal{P}_N = \left\{ W_p(\hat{\mathbf{P}}^{N}, \mathbf{P}) \leq \epsilon \right\}$$

Wasserstein distance



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**Data-driven Wasserstein DRO.** D. Kuhn, P. Mohajerin Esfahani, B. Van Parys, R. Gao, S. Shafieezadeh Abadeh, W. Wiesemann, F. Dörfler, L. Aolaritei, A. Selvi….
**Optimal Transport.** G. Monge, L. Kantorovic, L. Wasserstein, C. Villani, …

# Robust vs Distributionally Robust Optimization

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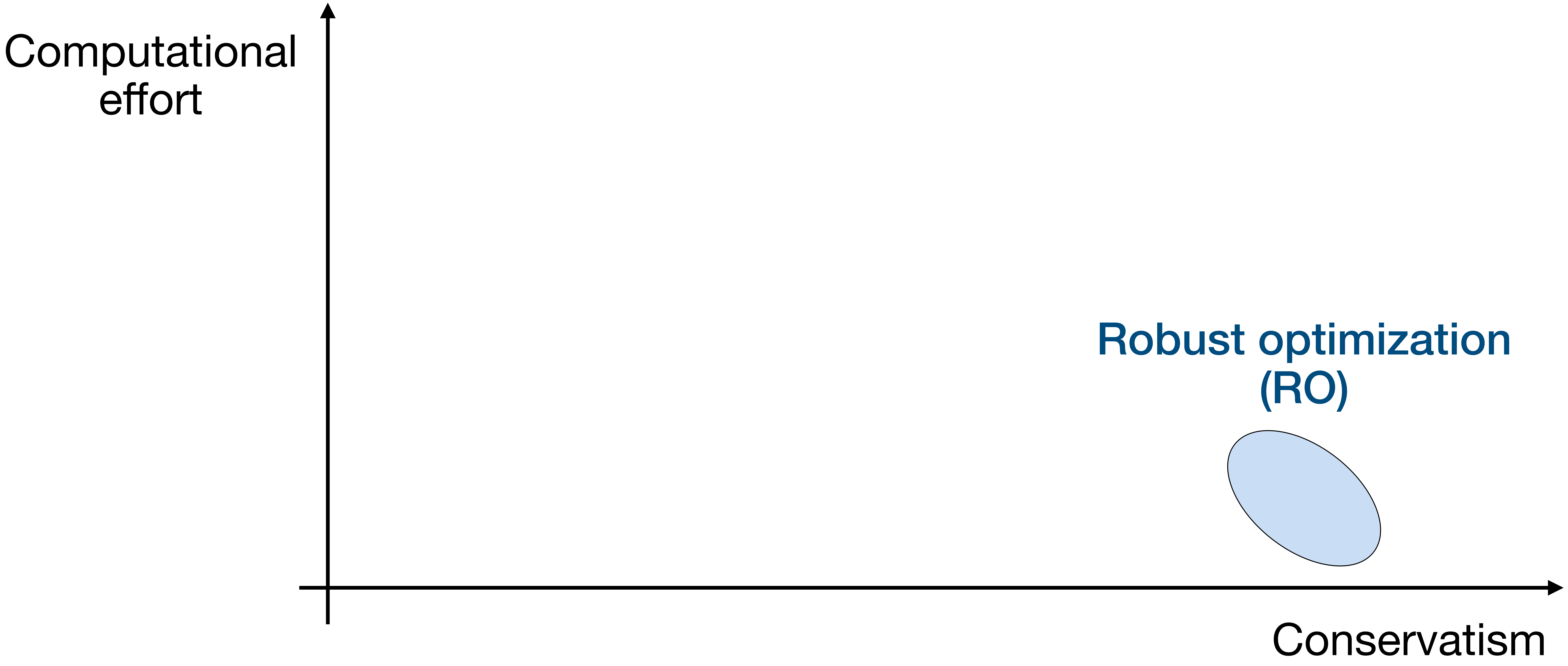

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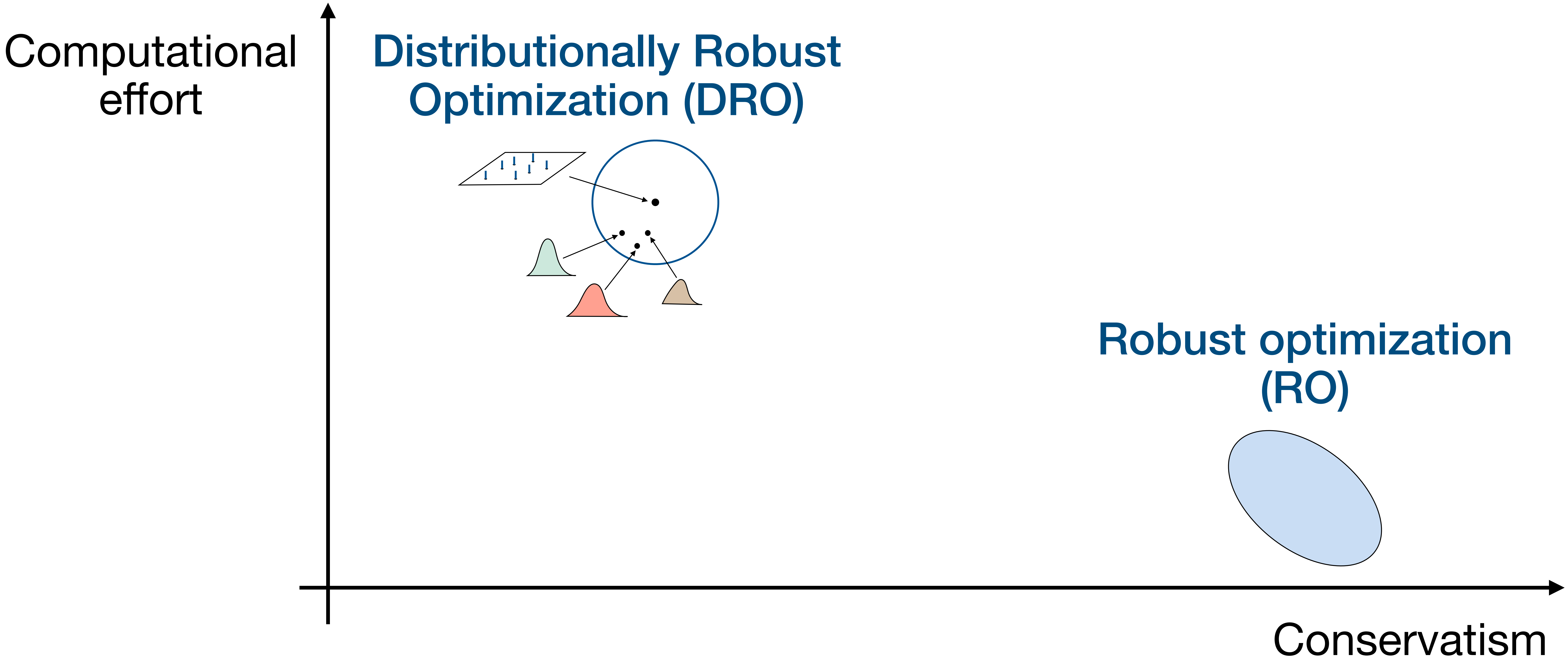


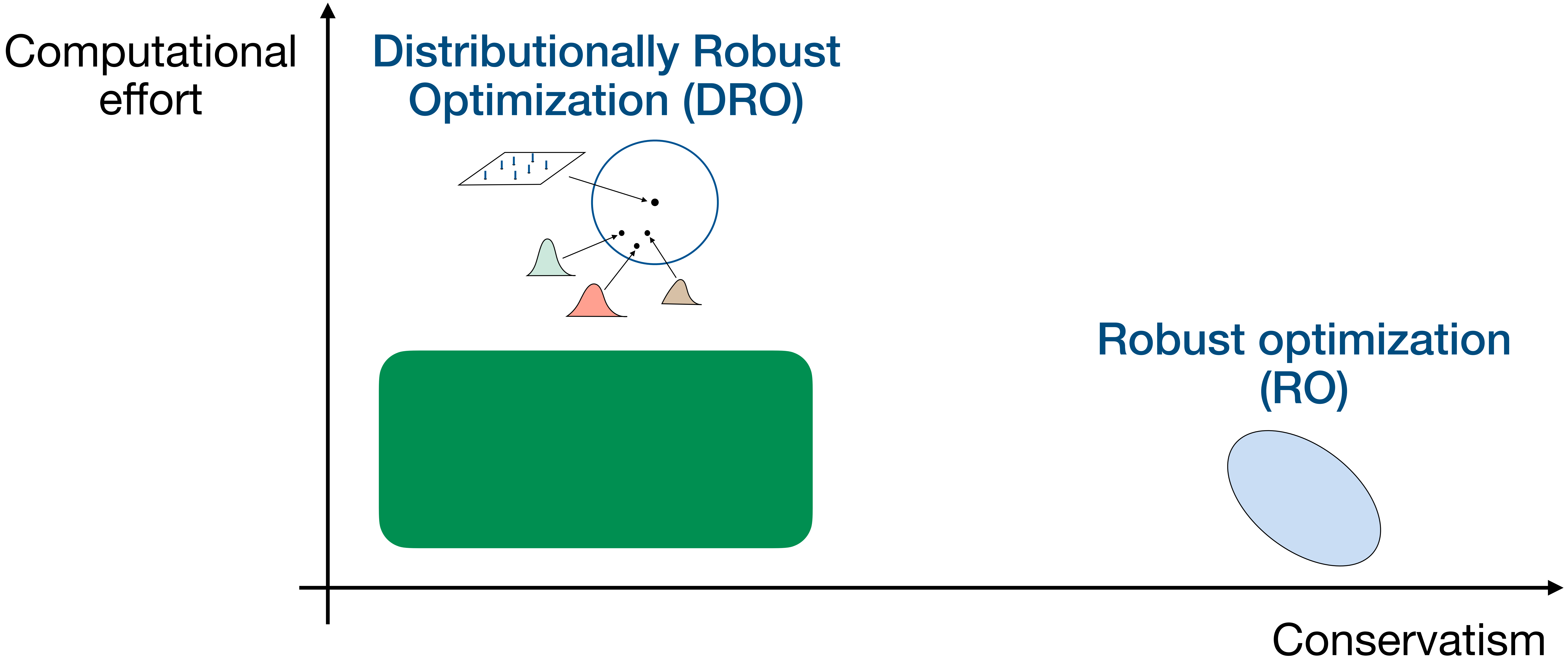
Computational effort

Distributionally Robust Optimization (DRO)

Robust optimization (RO)

Conservatism

# Robust vs Distributionally Robust Optimization



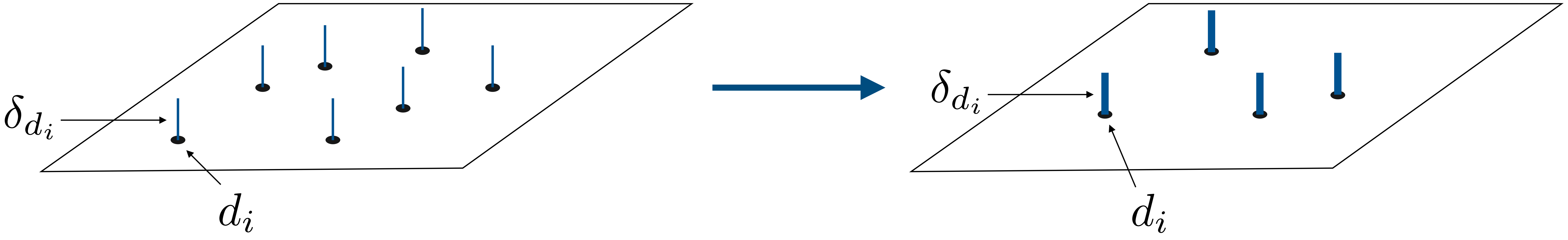
Can we get the best of both worlds?

# Mean Robust Optimization

# Data compression for decision-making under uncertainty

## scenario reduction

Dupačová et al. (2003), Beraldi and Bruni (2014),
Chen (2015), Emelogu et al. (2016), Jacobson et al. (2021)



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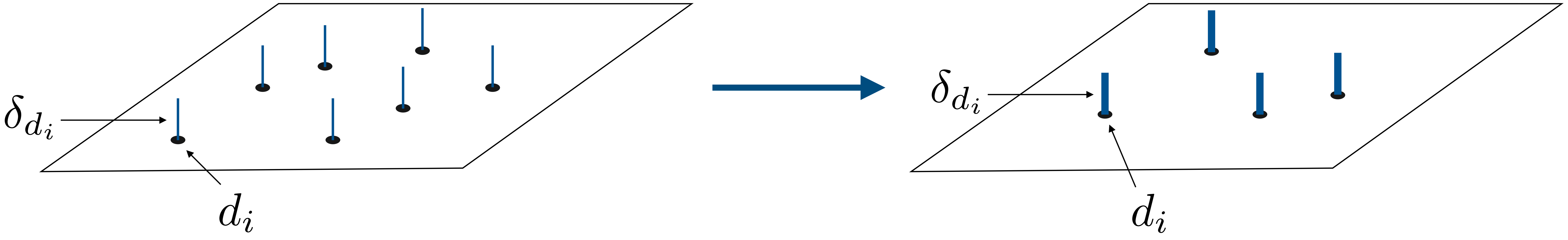

**approximation guarantees**

Rujeerapaiboon et al. (2022)

# Data compression for decision-making under uncertainty

## scenario reduction

Dupačová et al. (2003), Beraldi and Bruni (2014),
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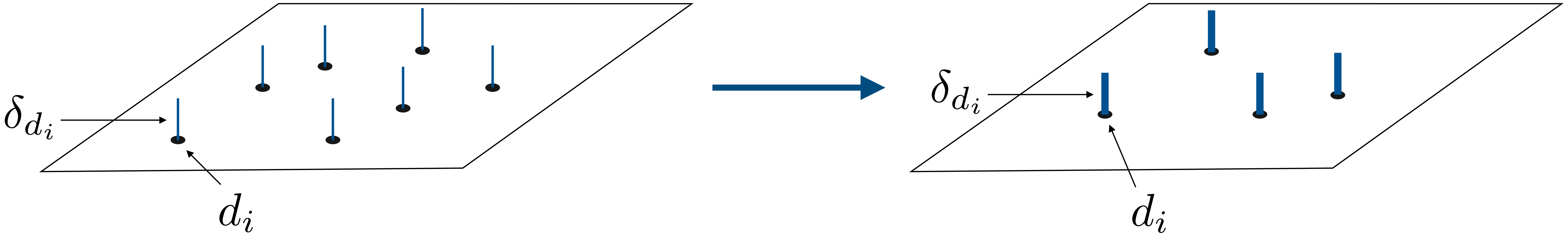

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## dynamic problems

**two-stage optimization**

Bertsimas and Mundru (2022)
Wang et al. (2023)

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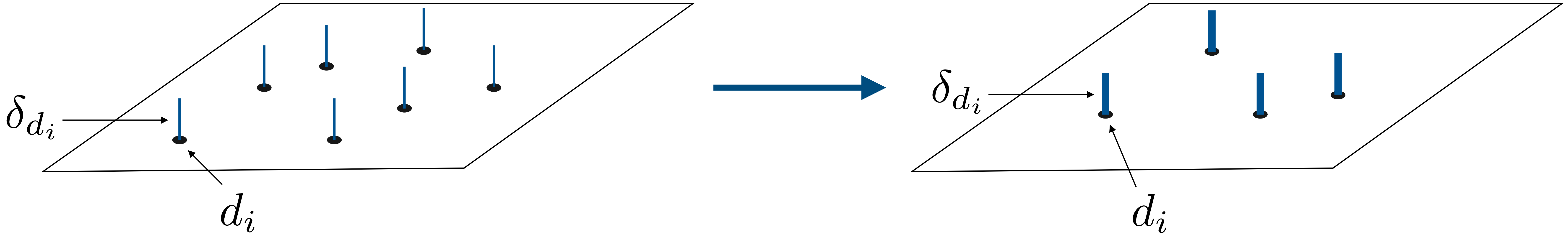

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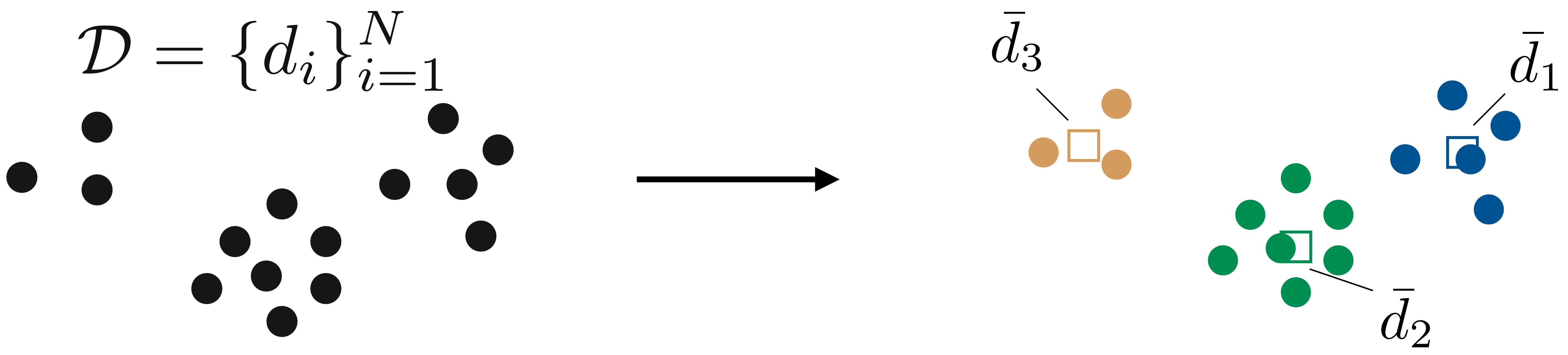
→ **tied to a specific clustering technique**

# We use clustering to reduce dimensionality and computation time

Data

$\mathcal{D} = \{d_i\}_{i=1}^{N}$



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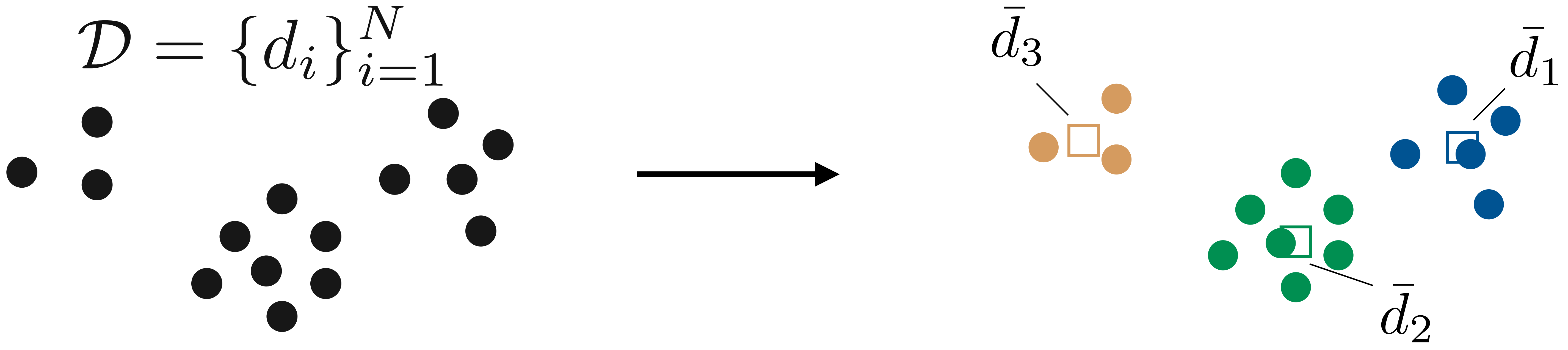

minimize $$\sum_{i=1}^{N} \sum_{k=1}^{K} a_{ik} \|d_i - \bar{d}_k\|^2$$

Jain et al. (1999)

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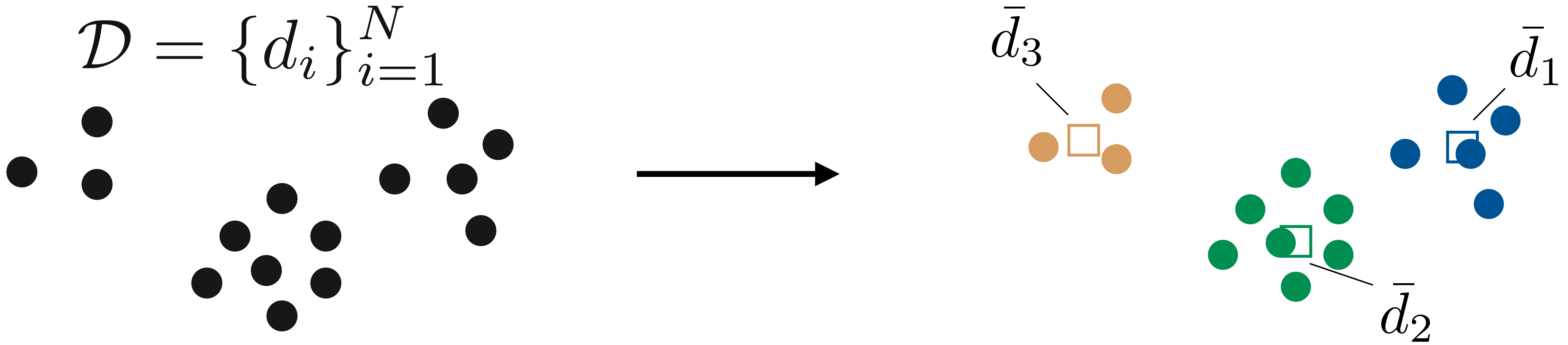

minimize $$\sum_{i=1}^{N}\sum_{k=1}^{K} \underbrace{a_{ik}}_{\text{cluster assignments}} \|d_i - \bar{d}_k\|^2$$

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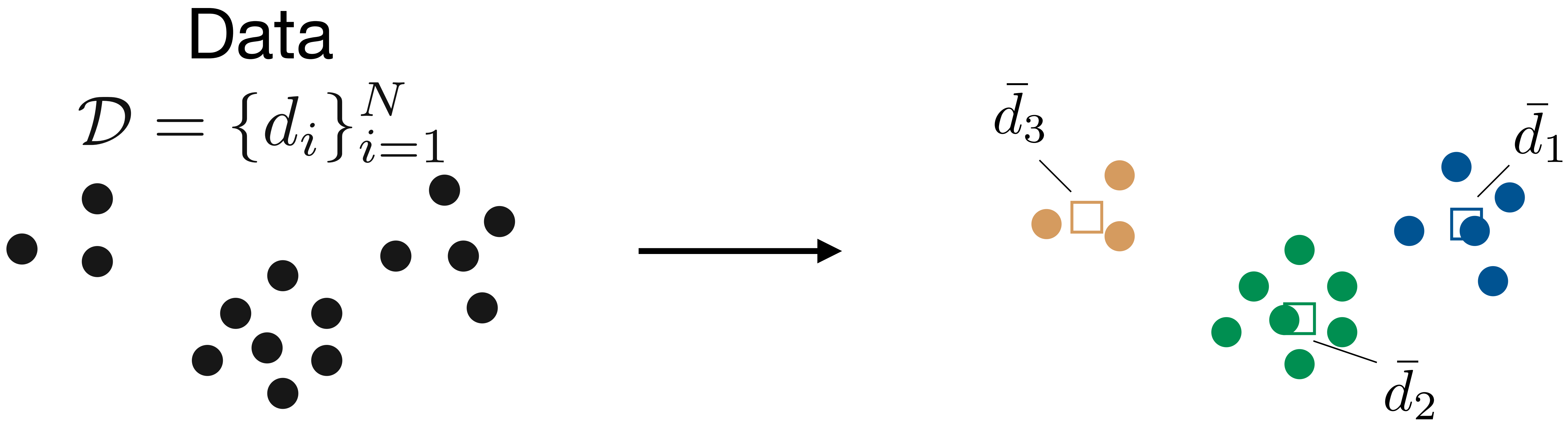
# We use clustering to reduce dimensionality and computation time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cluster assignments

cluster centers

Jain et al. (1999)

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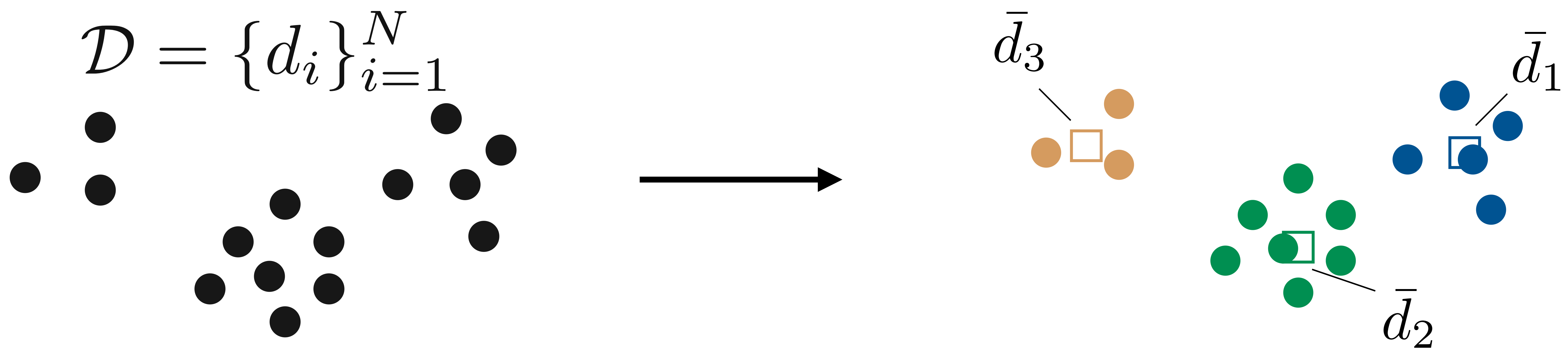

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*we work with any clustering algorithm*

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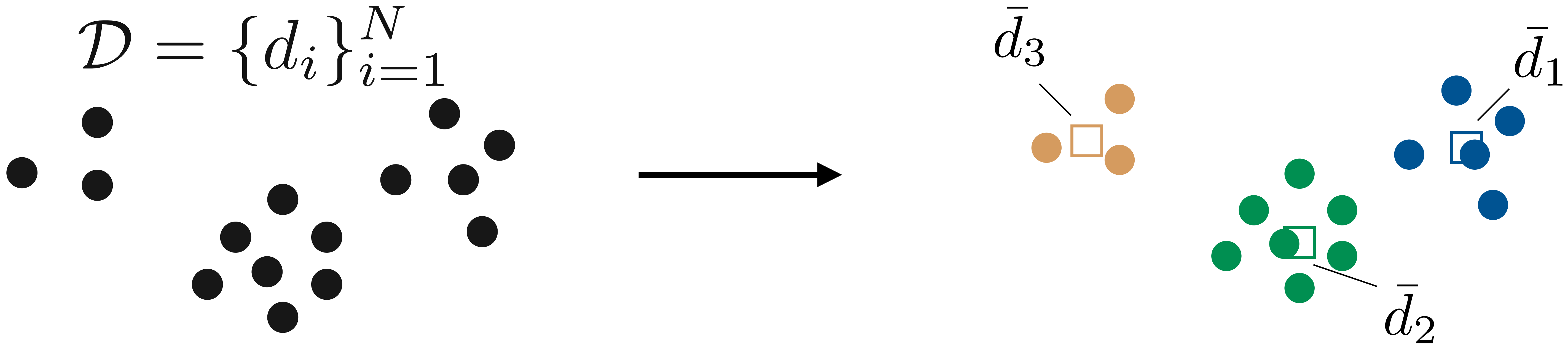

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cluster assignments

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## Main idea

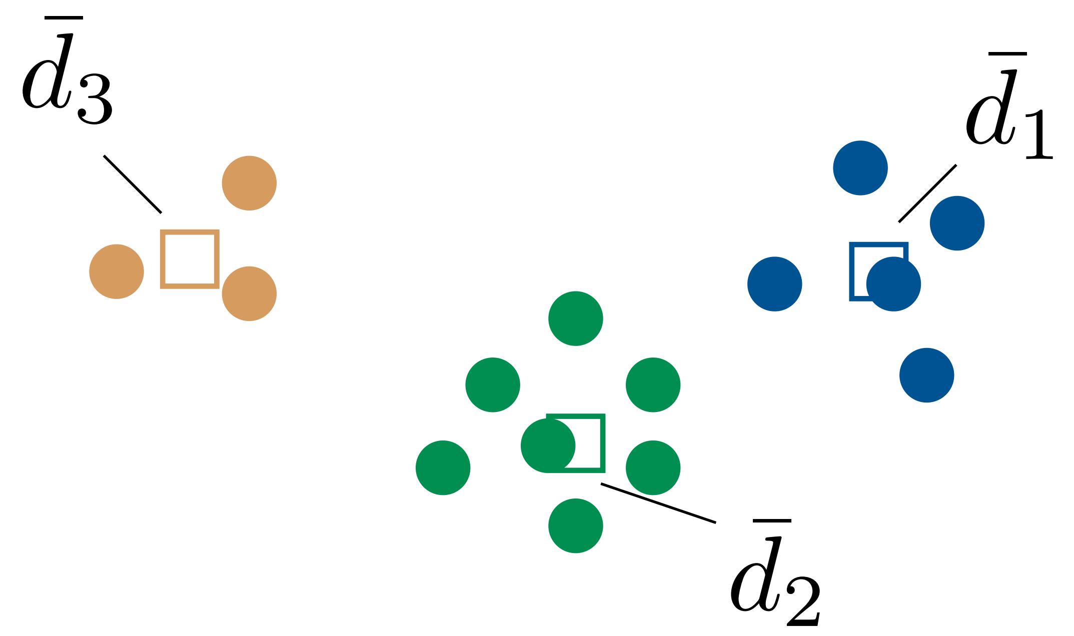
use cluster centers
instead of original data

Jain et al. (1999)

# Our approach: Mean Robust Optimization (MRO)

Data

$\mathcal{D}_N \longrightarrow$ **Clustering**



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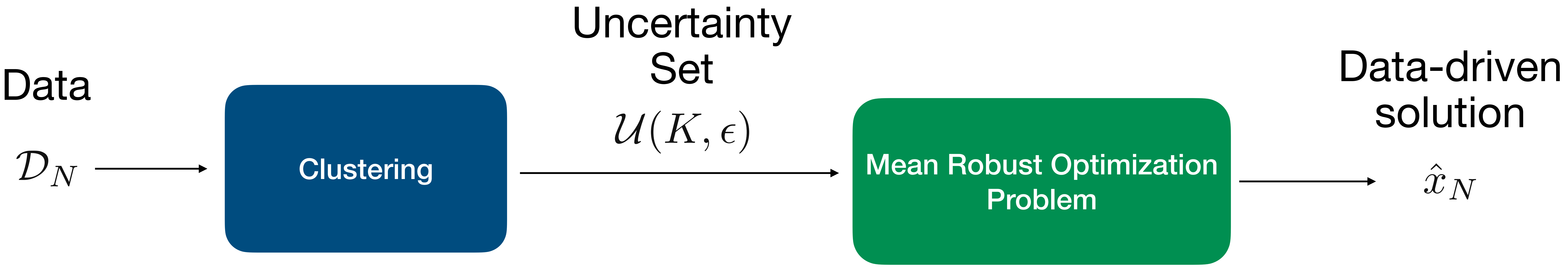


Data

$\mathcal{D}_N$

Clustering

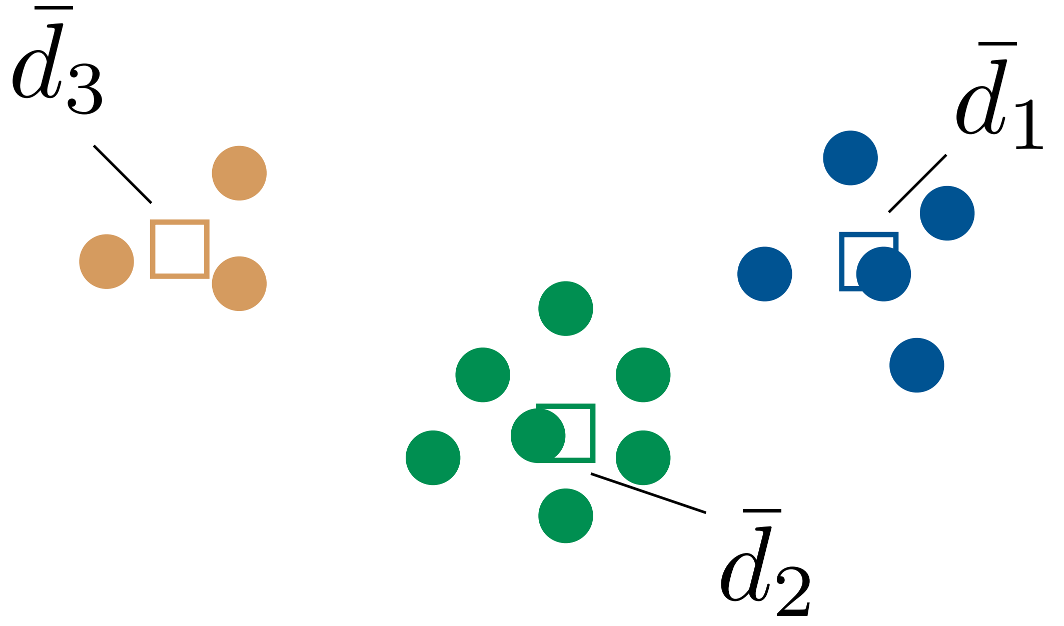
Uncertainty Set

$\mathcal{U}(K, \epsilon)$

Mean Robust Optimization Problem

Data-driven solution

$\hat{x}_N$



$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & \bar{g}(x, u) \leq 0 \quad \forall u \in \mathcal{U}(K, \epsilon) \end{array}$$

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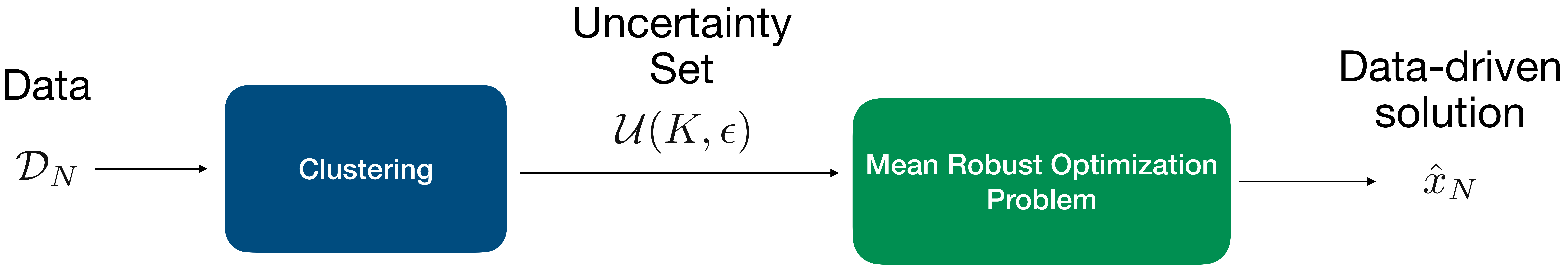

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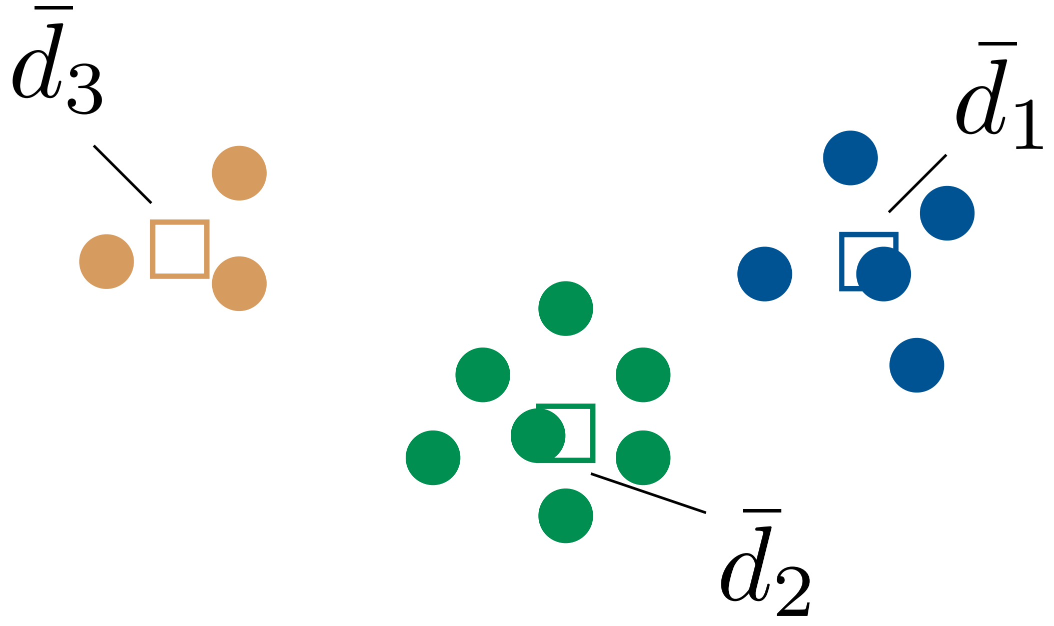

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$\uparrow$

constraint function

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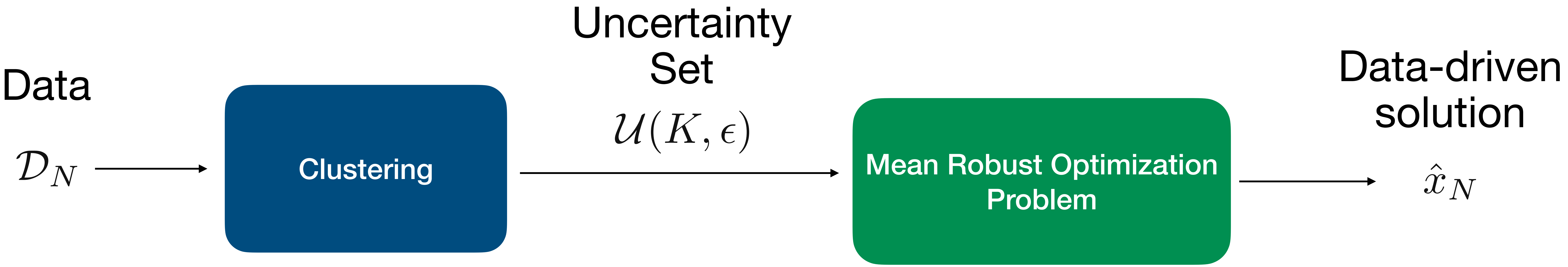

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Mean Robust Optimization Problem

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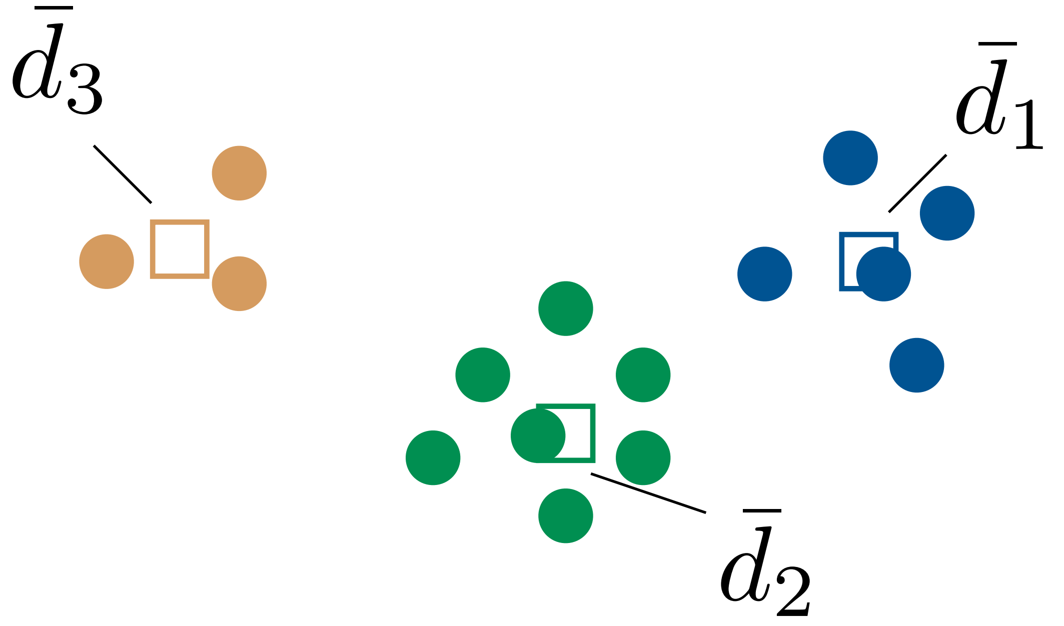

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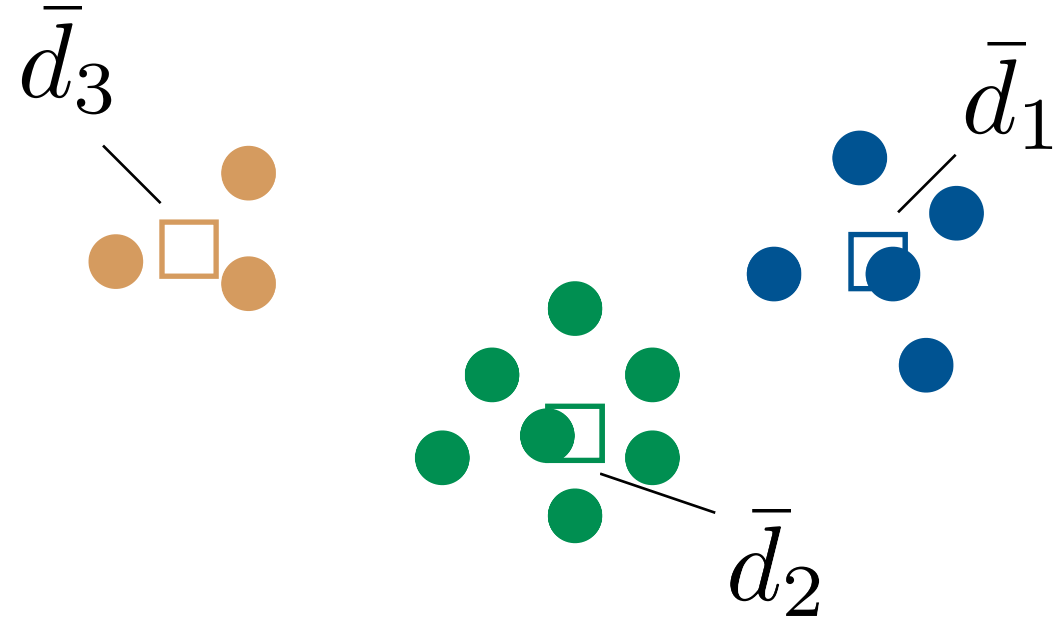
constraint function

uncertainty set

# Uncertainty set



$$\mathcal{U}(K,\epsilon) = \left\{ u = (v_1,\ldots,v_K) \;\middle|\; \sum_{k=1}^{K} w_k \|v_k - \bar{d}_k\|^p \le \epsilon^p \right\}$$

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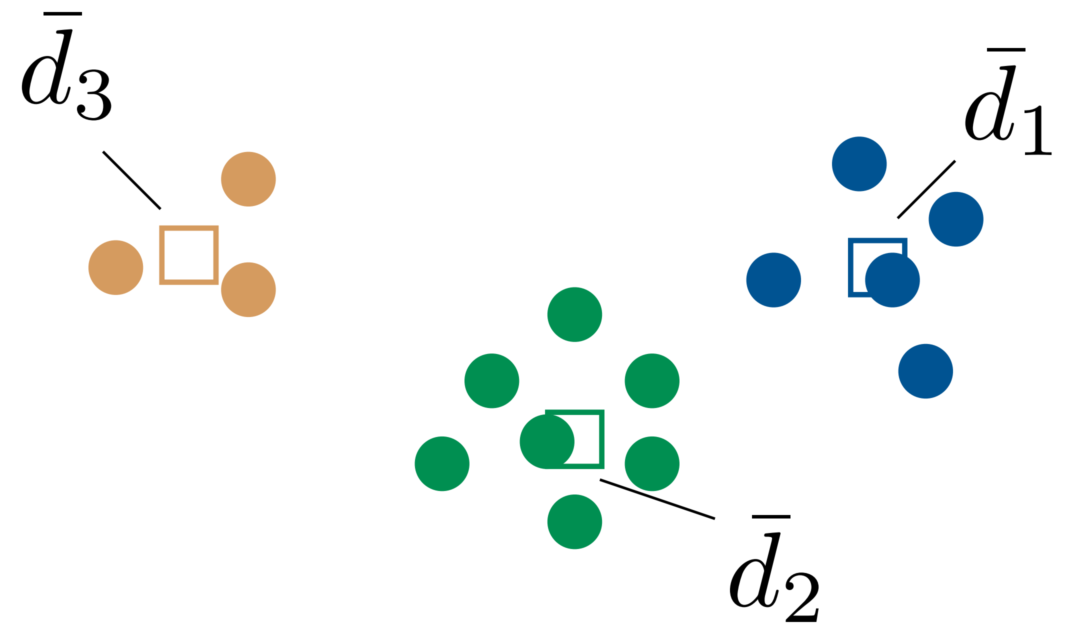

cluster weights

$$\mathcal{U}(K,\epsilon) = \left\{ u = (v_1, \ldots, v_K) \;\middle|\; \sum_{k=1}^{K} w_k \|v_k - \bar{d}_k\|^p \leq \epsilon^p \right\}$$

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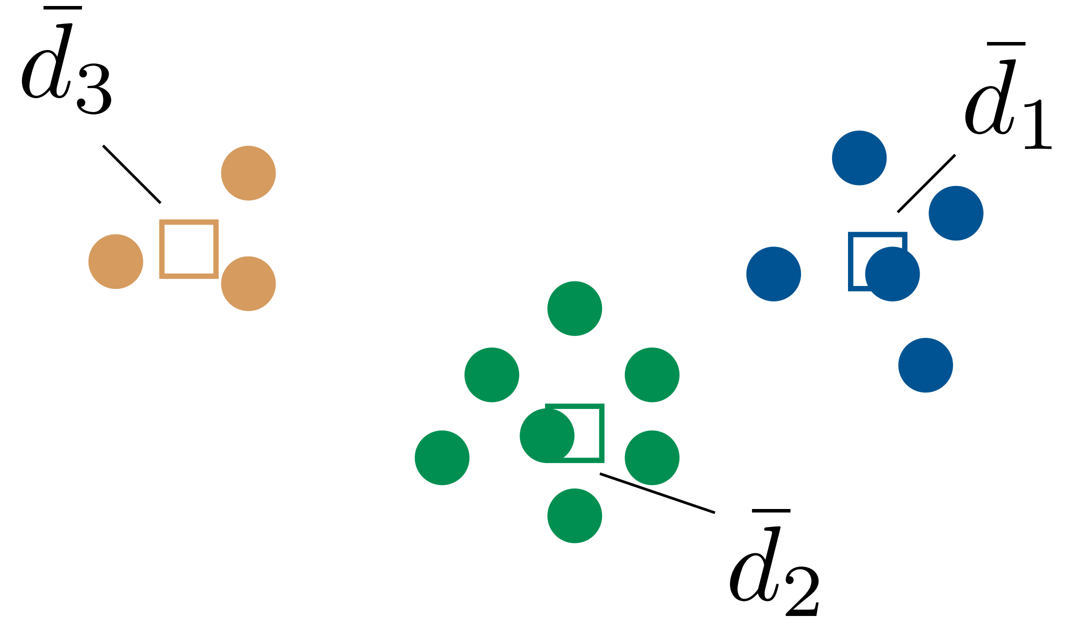

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cluster centers

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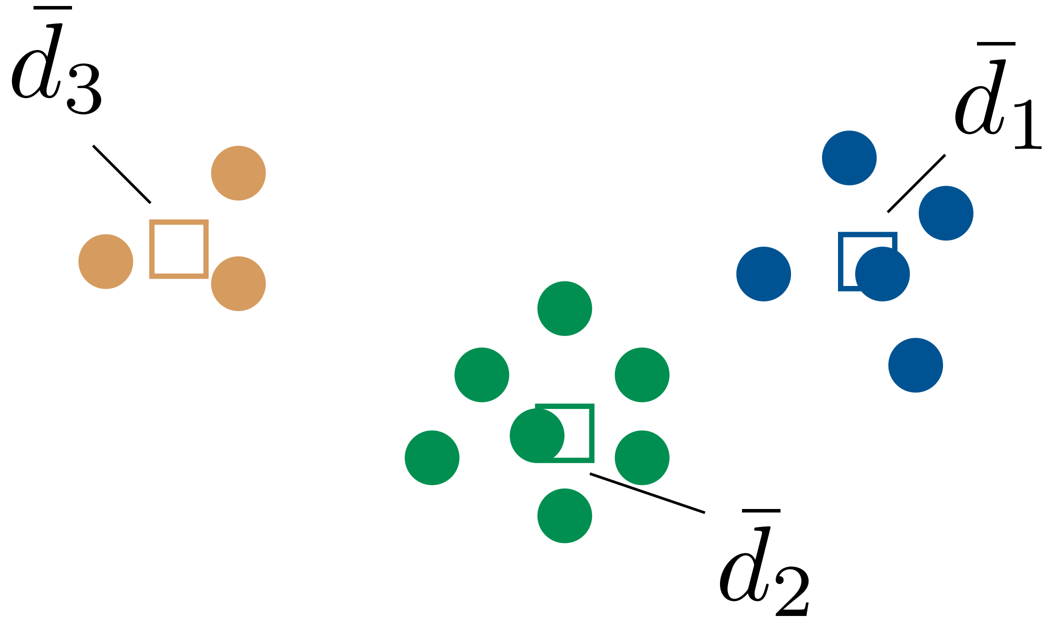

cluster weights

order

$$\mathcal{U}(K, \epsilon) = \left\{ u = (v_1, \dots, v_K) \;\middle|\; \sum_{k=1}^{K} w_k \|v_k - \bar{d}_k\|^p \le \epsilon^p \right\}$$

cluster centers

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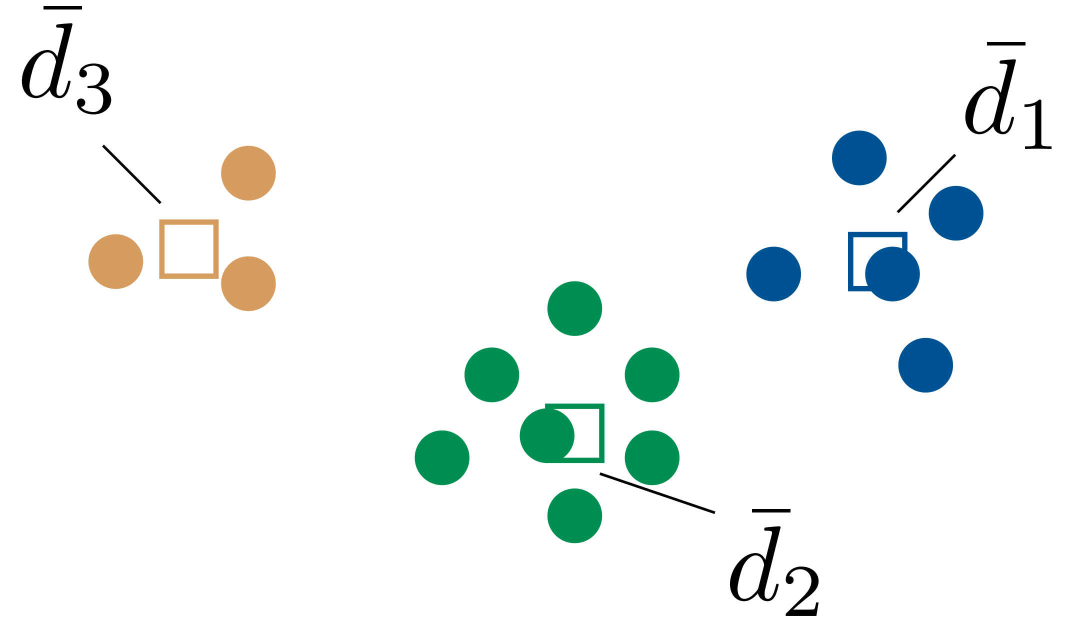

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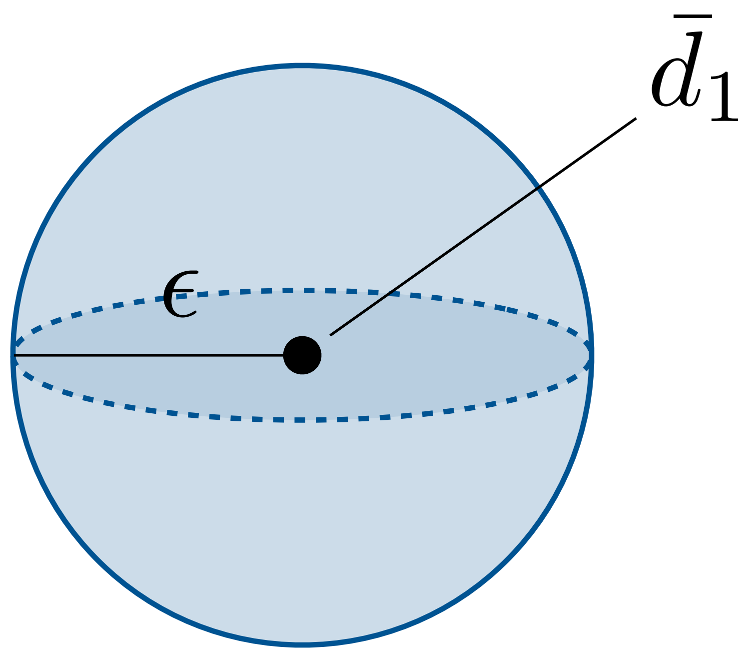
cluster centers

**Examples**

$K = 1$



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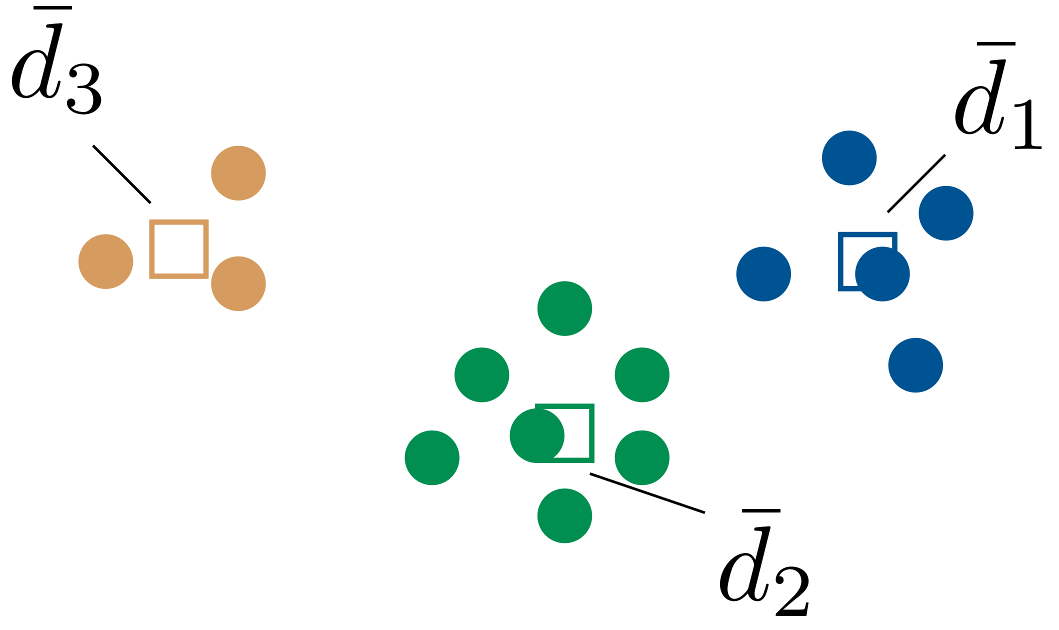

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$K$

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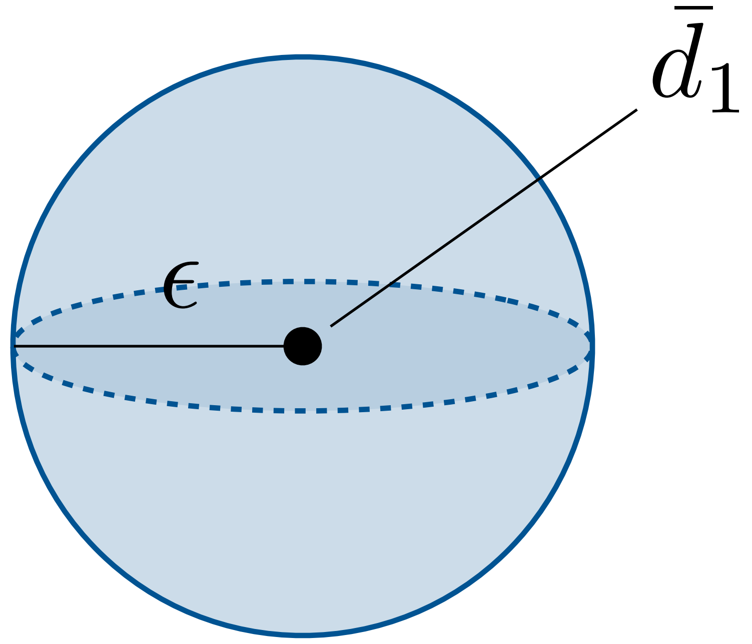
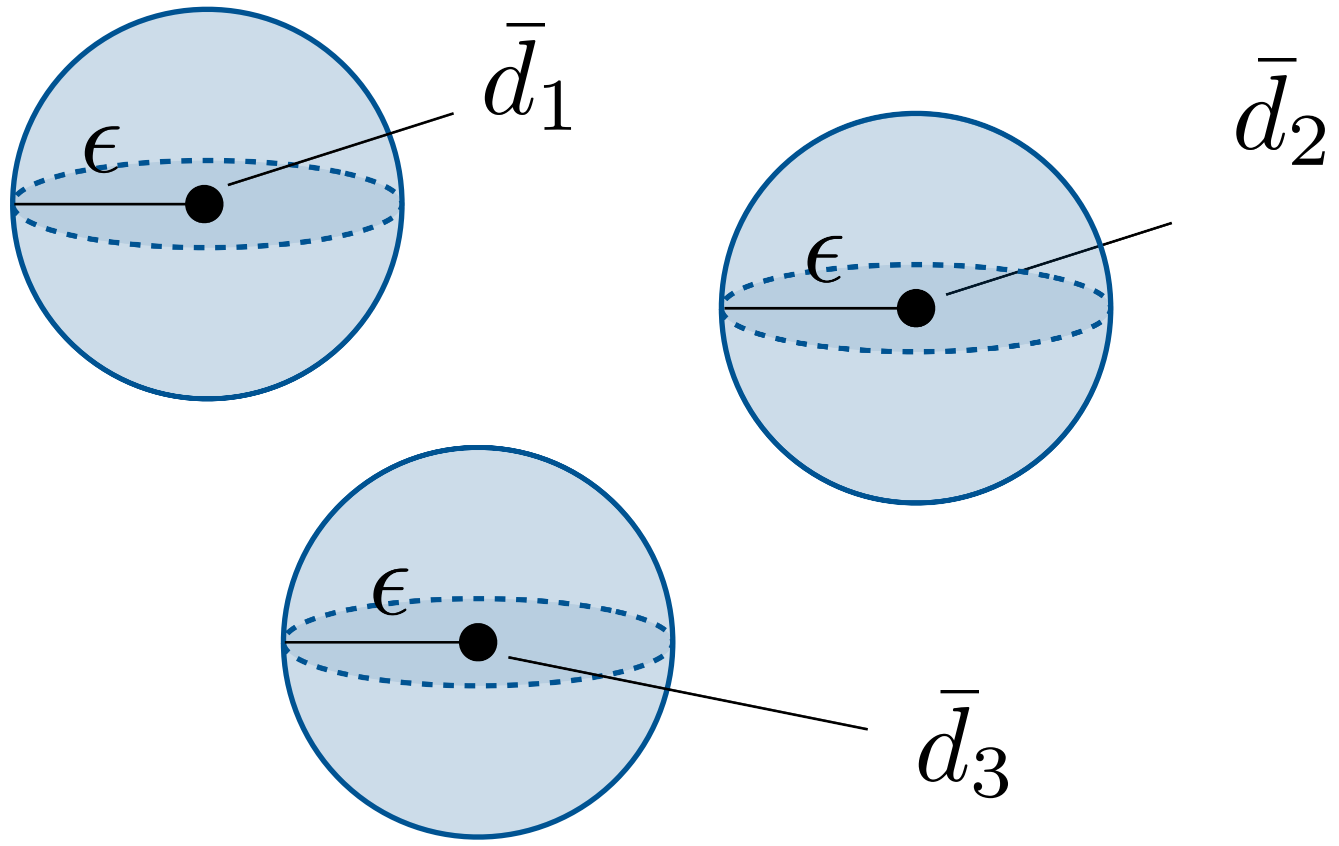


$K = 3, p = \infty$



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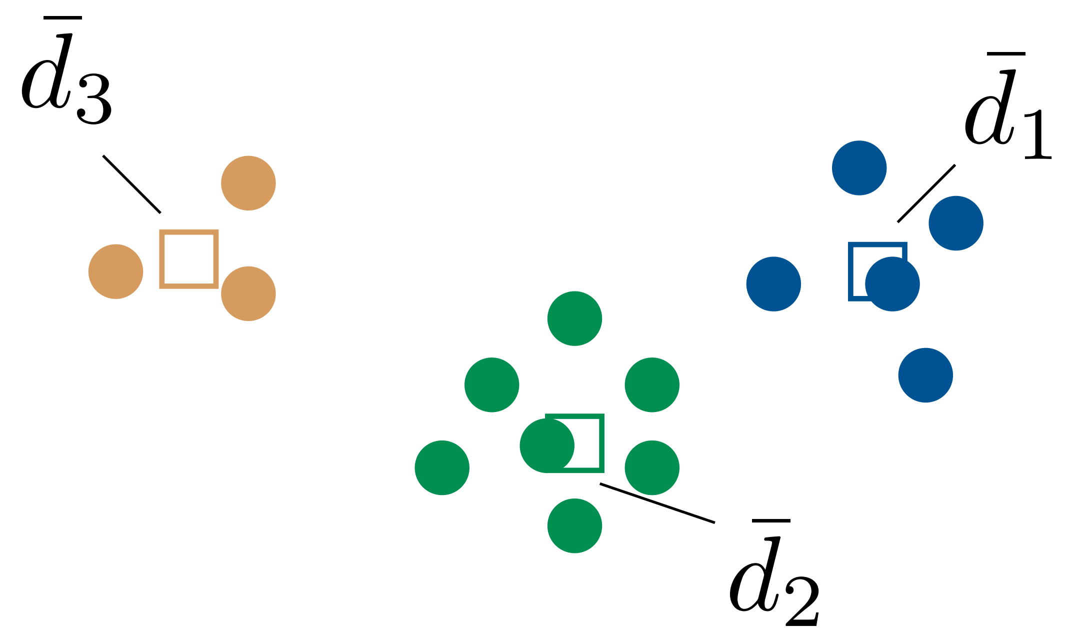

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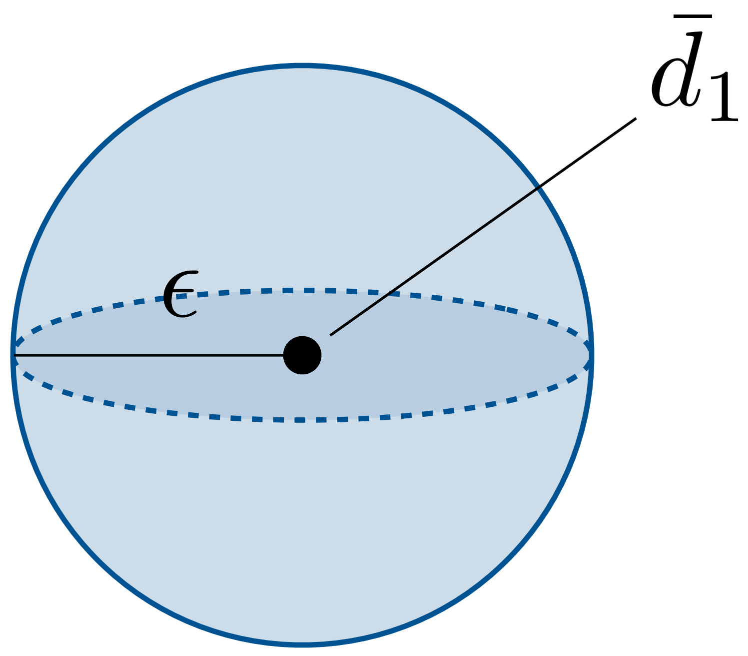

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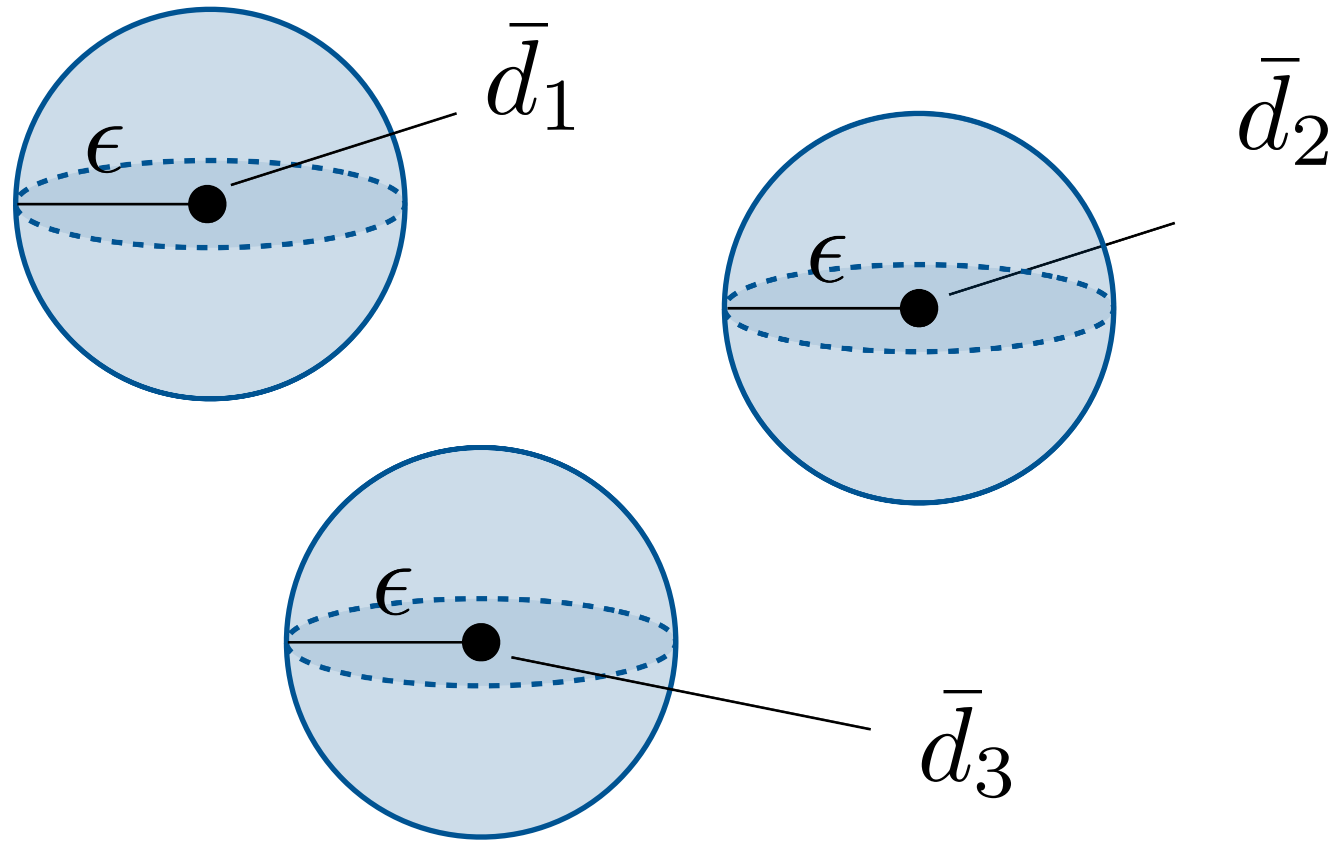
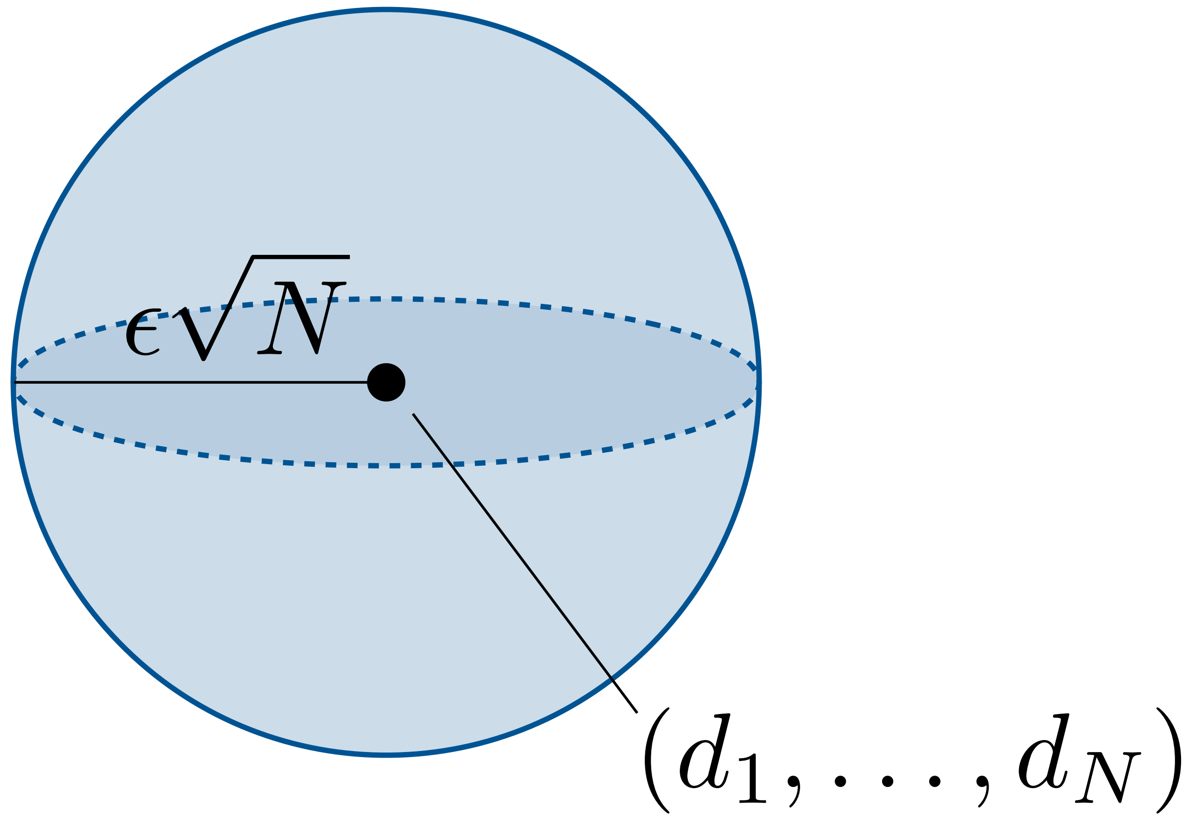


$K = N, p = 2$



# Back to the Mean Robust Optimization problem

Uncertain variable lifting

$u = (v_1, \dots, v_K)$

$$\longrightarrow$$

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & \bar{g}(x,u) \le 0 \quad \forall u \in \mathcal{U}(K,\epsilon) \end{array}$$

# Back to the Mean Robust Optimization problem

uncertainty set

$$\left\{ \sum_{k=1}^{K} w_k \|v_k - \bar{d}_k\|^p \leq \epsilon^p \right\}$$

Uncertain variable lifting

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Uncertain variable lifting

$u = (v_1, \dots, v_K)$

$\longrightarrow$

$$
\begin{array}{ll}
\text{minimize} & f(x) \\
\text{subject to} & \bar{g}(x, u) \le 0 \quad \forall u \in \mathcal{U}(K, \epsilon)
\end{array}
$$

uncertainty set

$$
\left\{ \sum_{k=1}^{K} w_k \|v_k - \bar{d}_k\|^p \le \epsilon^p \right\}
$$

constraint function

$$
\sum_{k=1}^{K} w_k g(x, v_k)
$$

# Solving the MRO problem

dualize constraint

$$\bar{g}(x,u) \leq 0, \ \forall u \in \mathcal{U}(K,\epsilon)$$

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conjugate function

cluster centers

function of $p \geq 1$
$\phi(p) \rightarrow 1$ as $p \rightarrow \infty$
$\phi(1) = 0$

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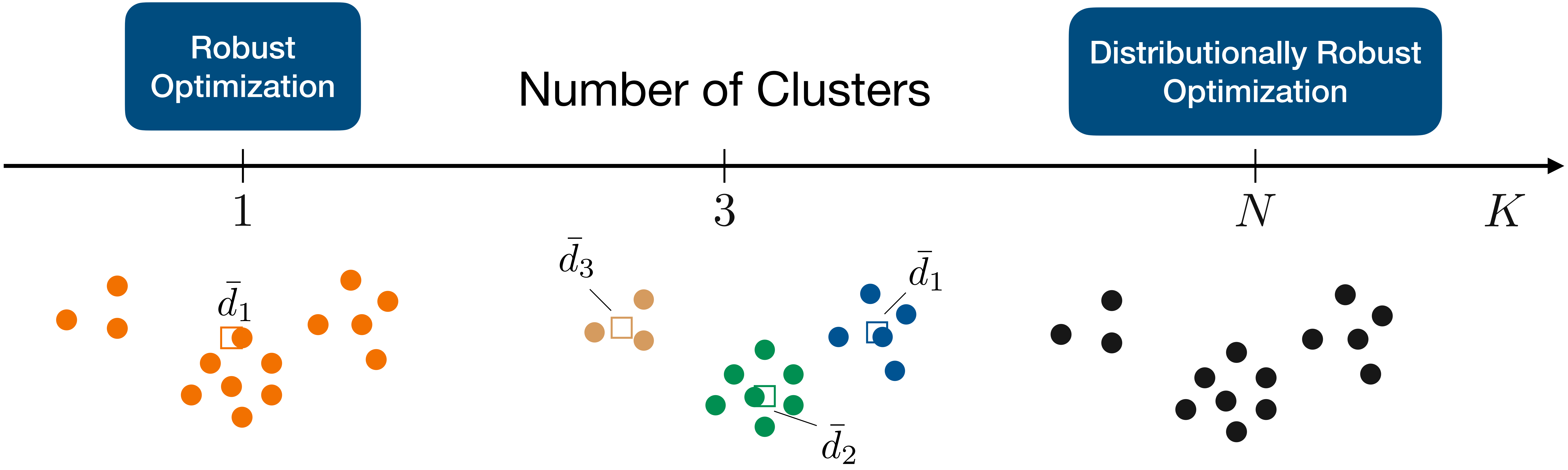
function of $p \geq 1$

$\phi(p) \to 1$ as $p \to \infty$

$\phi(1) = 0$

It can be very expensive when $K$ is large (e.g., $K = N$)

# MRO bridges RO and DRO



**Robust Optimization**

Number of Clusters

**Distributionally Robust Optimization**

Kuhn, Mohajerin Esfahani, Nguyen, Shafieezadeh-Abadeh (2019)

# Probabilistic guarantees in MRO

$$\mathbf{P}^N\left(\mathbf{E}(g(\hat{x}_N, u)) \leq 0\right) \geq 1 - \beta$$

# Probabilistic guarantees in MRO

probability of
constraint
satisfaction

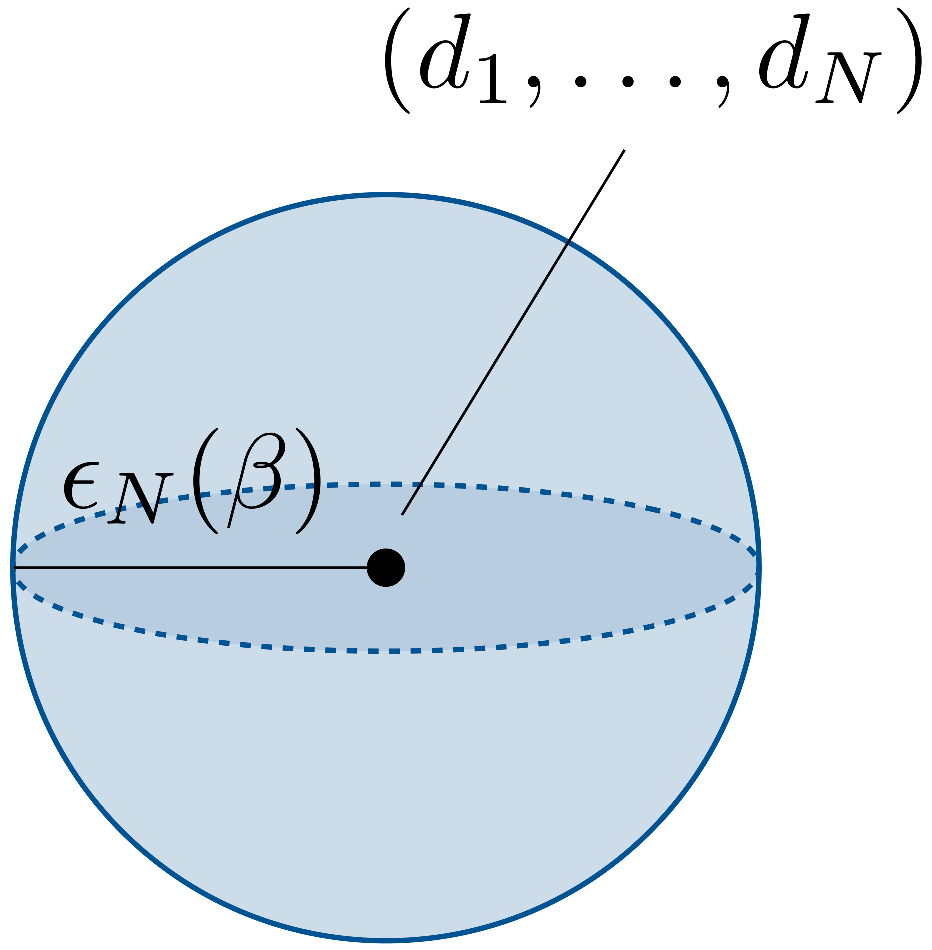
$$\mathbf{P}^N\left(\mathbf{E}(g(\hat{x}_N, u)) \leq 0\right) \geq \boxed{1-\beta}$$

# Probabilistic guarantees in MRO

probability of constraint satisfaction

uncertainty set radius

$$\mathbf{P}^N\left(\mathbf{E}(g(\hat{x}_N,u)) \le 0\right) \ge 1-\beta \quad \xrightarrow[\text{Fournier and Guillin (2013)}]{\text{light-tailed}} \quad \mathcal{U}(N,\epsilon_N(\beta))$$



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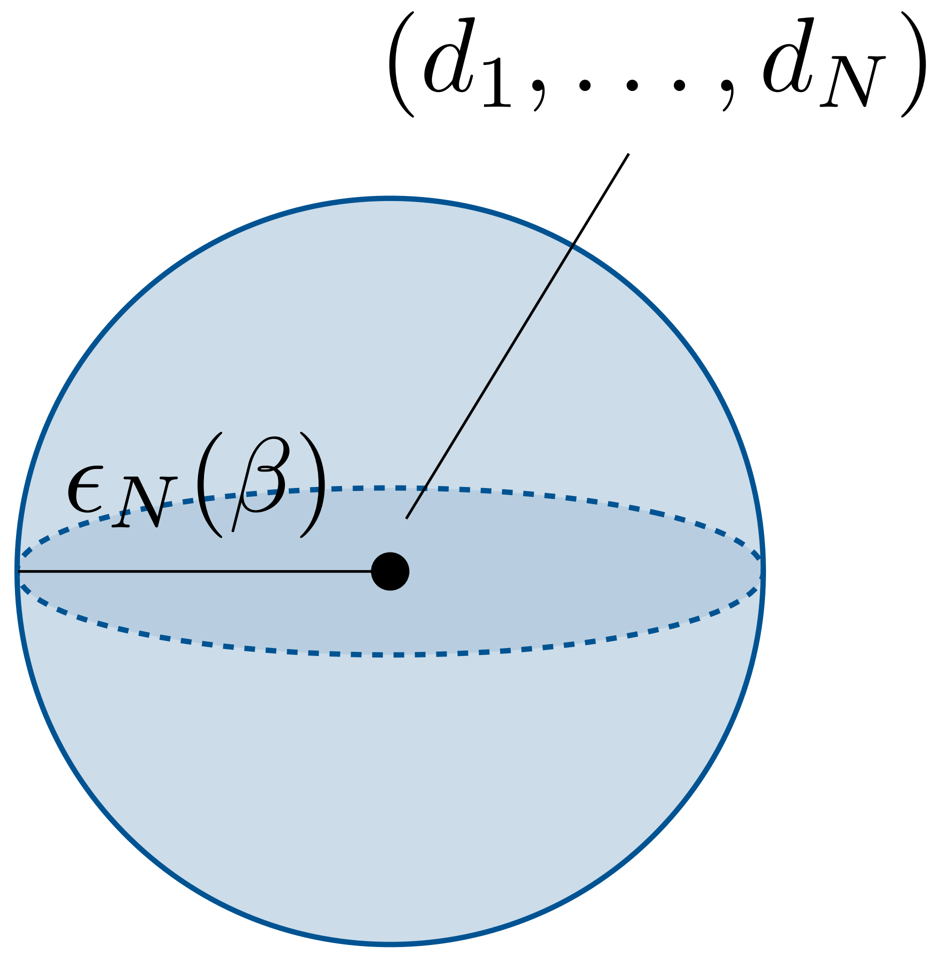
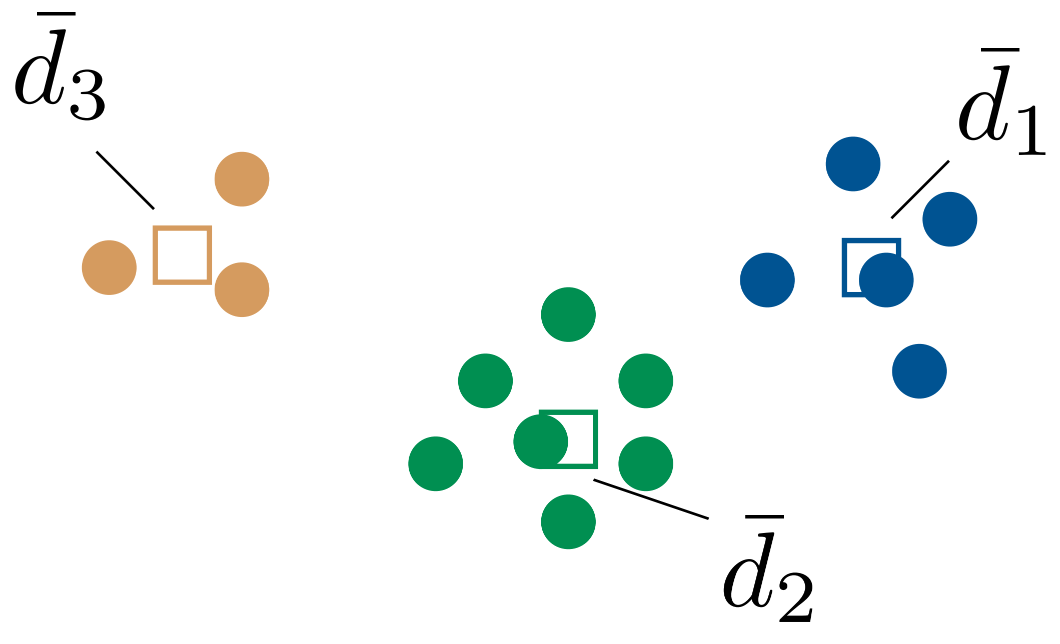


**MRO clustering**

$$\mathcal{U}(K,\epsilon_N(\beta)+\eta_K)$$

clustering error

$$\sum_{i=1}^{N}\sum_{k=1}^{K} a_{ik}\|d_i-\bar{d}_k\|^p$$



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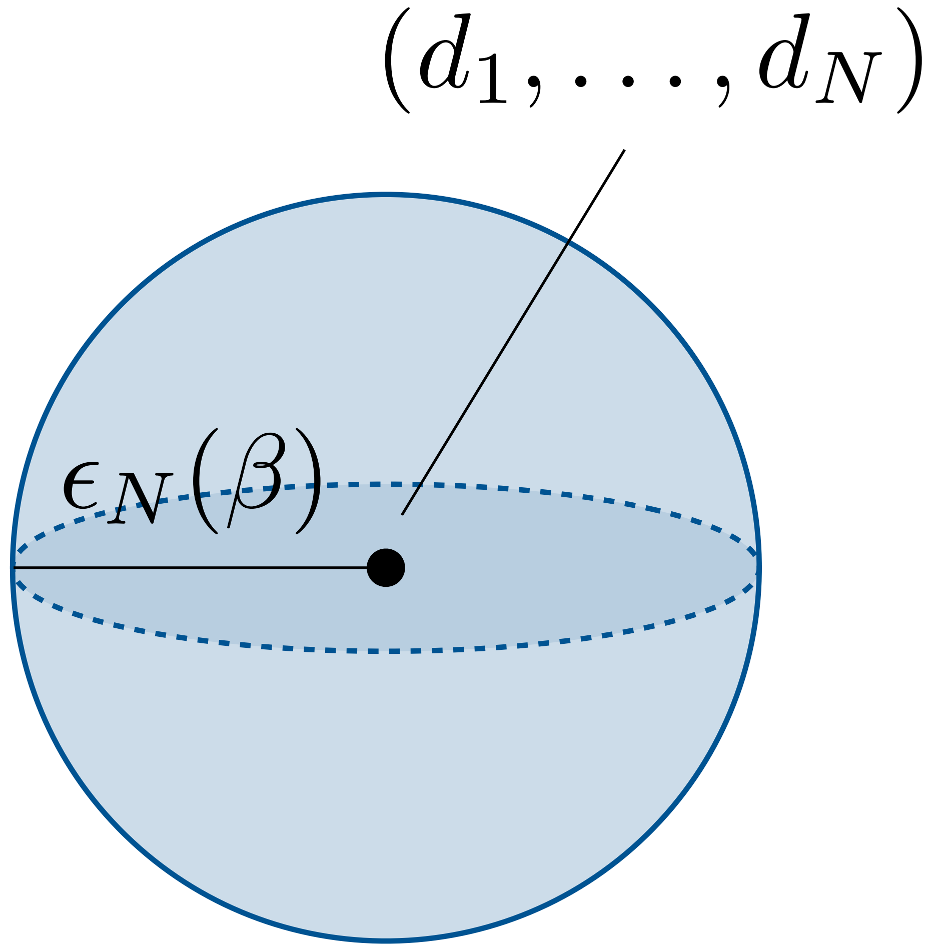

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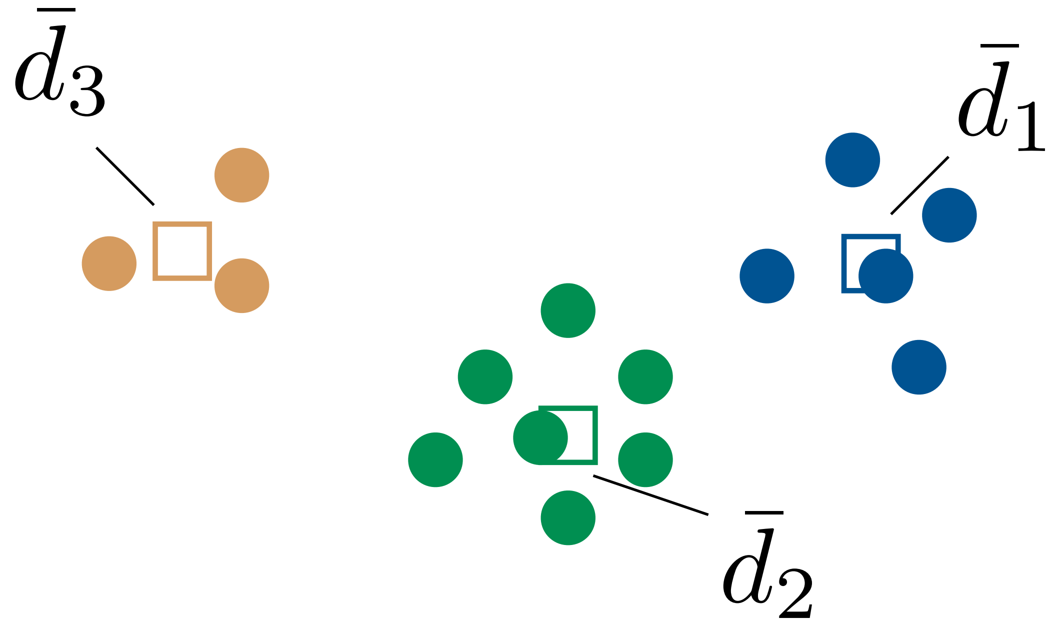

Quite conservative bounds… can we do better?

# Bounding the conservatism

## MRO constraint

$$\bar{g}(x,u) \leq 0 \quad \forall u \in \mathcal{U}(K,\epsilon)$$

## Worst-case values

$$\bar{g}^{N}(x) = \underset{u \in \mathcal{U}(N,\epsilon)}{\text{maximize}} \quad \bar{g}(x,u)$$

$$\bar{g}^{K}(x) = \underset{u \in \mathcal{U}(K,\epsilon)}{\text{maximize}} \quad \bar{g}(x,u)$$

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### Theorem

If $-g$ is $L$-smooth in $u$, we have

$$\bar{g}^{N}(x) \leq \bar{g}^{K}(x) \leq \bar{g}^{N}(x) + \frac{L}{2} D(K)$$

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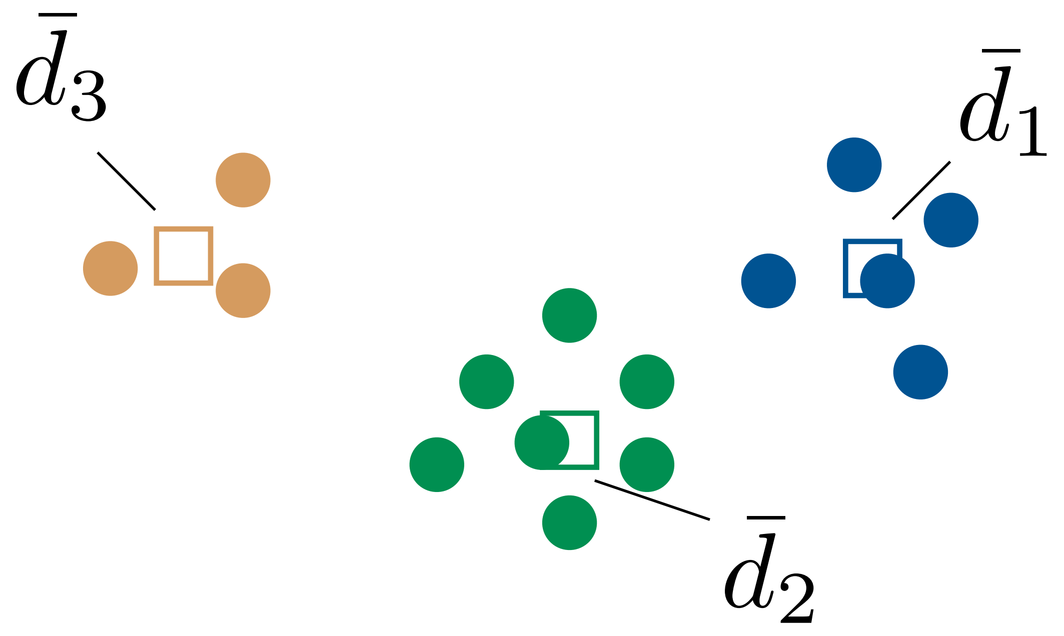
## clustering value

$$\sum_{i=1}^{N}\sum_{k=1}^{K} a_{ik}\|d_i - \bar{d}_k\|^2$$



# Bounding the conservatism

**Worst-case values**

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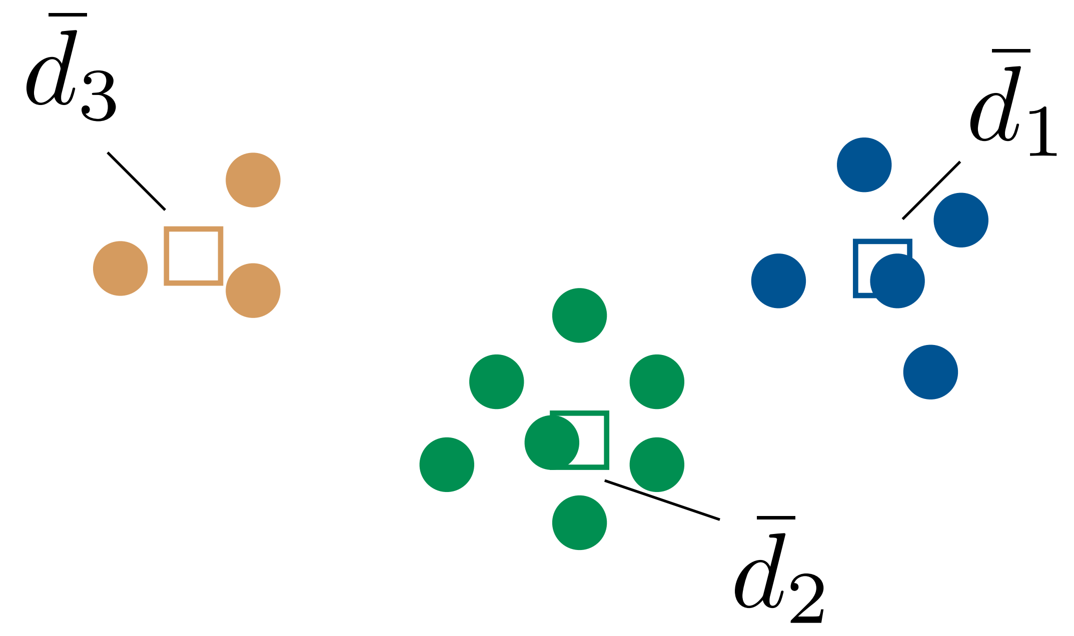
clustering value

$$\sum_{i=1}^{N} \sum_{k=1}^{K} a_{ik} \|d_i - \bar{d}_k\|^2$$



**When $g$ is affine in $u$ ($L = 0$), clustering makes no difference to the optimal value or optimal solution**

# We can also model maximum-of-concave constraints

$$g(x, u) = \max_{j \leq J} g_j(x, u)$$

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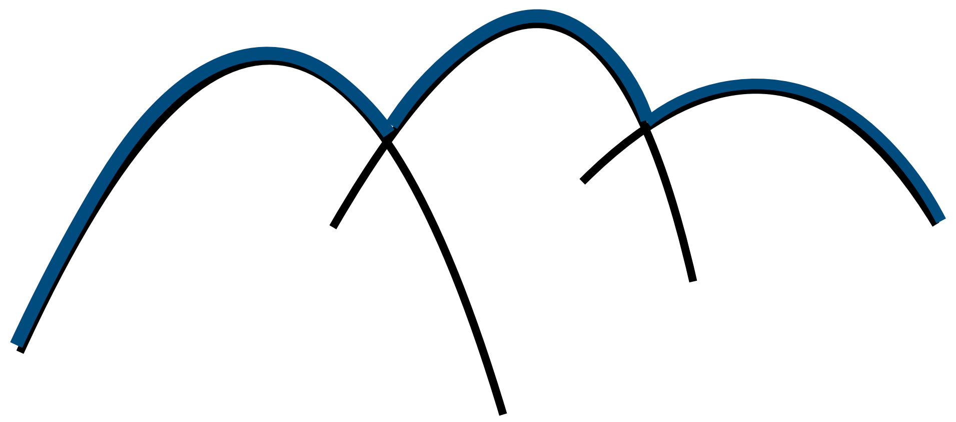

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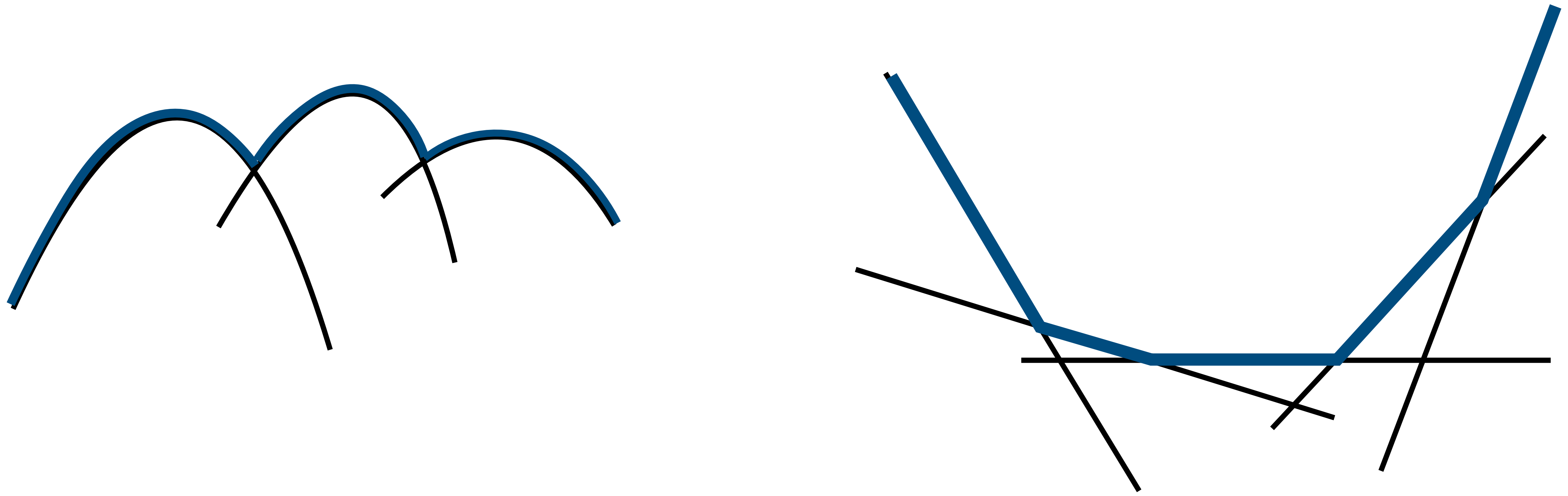

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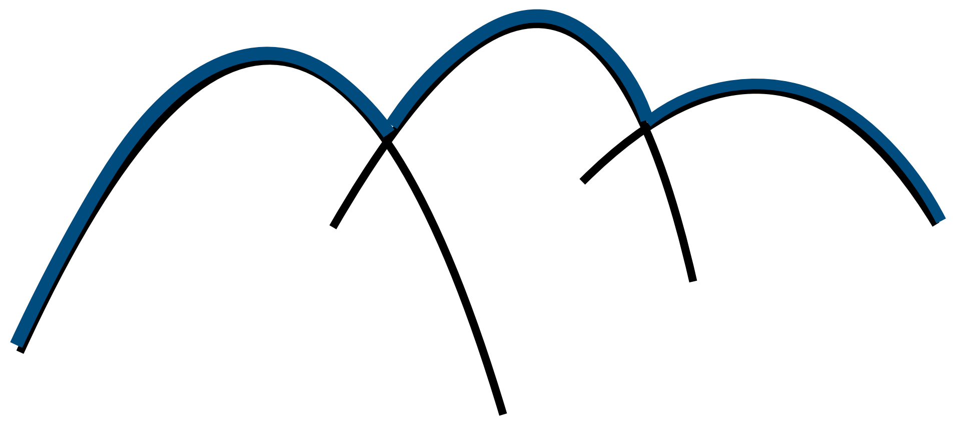

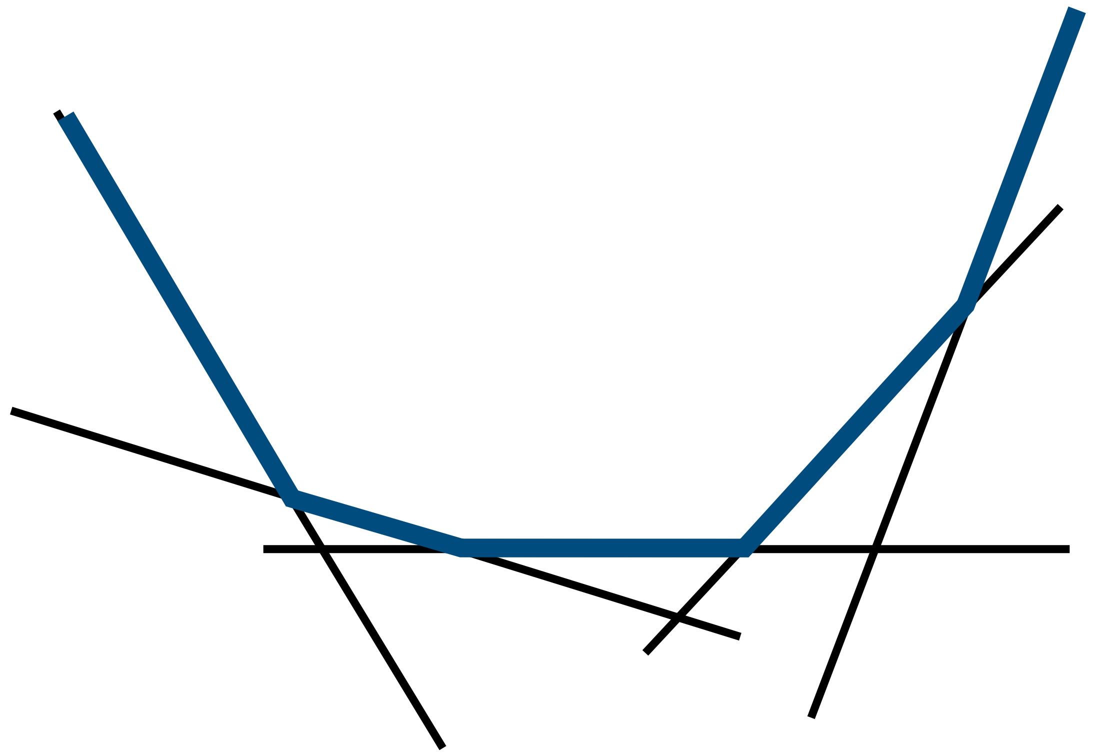

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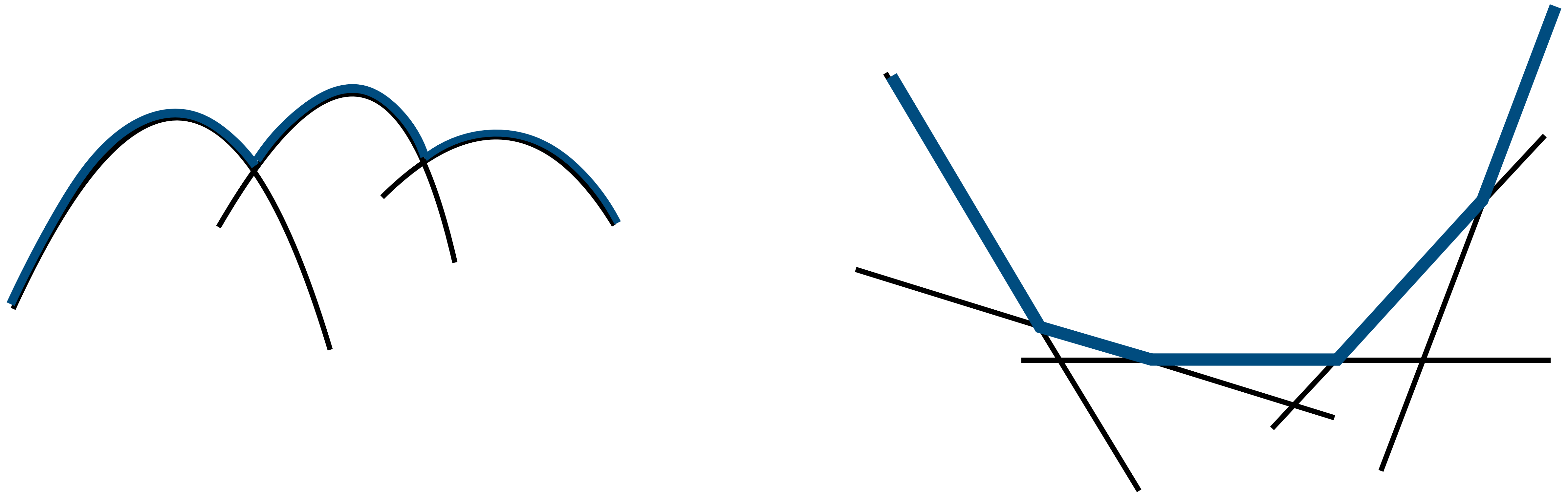

## MRO uncertainty set

$$\mathcal{U}(K,\epsilon) = \left\{ u = (v_{11},\dots,v_{JK}) \in \mathbf{R}^{J\times K} \;\middle|\; \exists \alpha \in \Gamma,\;\; \sum_{k=1}^{K}\sum_{j=1}^{J} \alpha_{jk}\|v_{jk} - \bar{d}_k\|^p \le \epsilon^p \right\}$$

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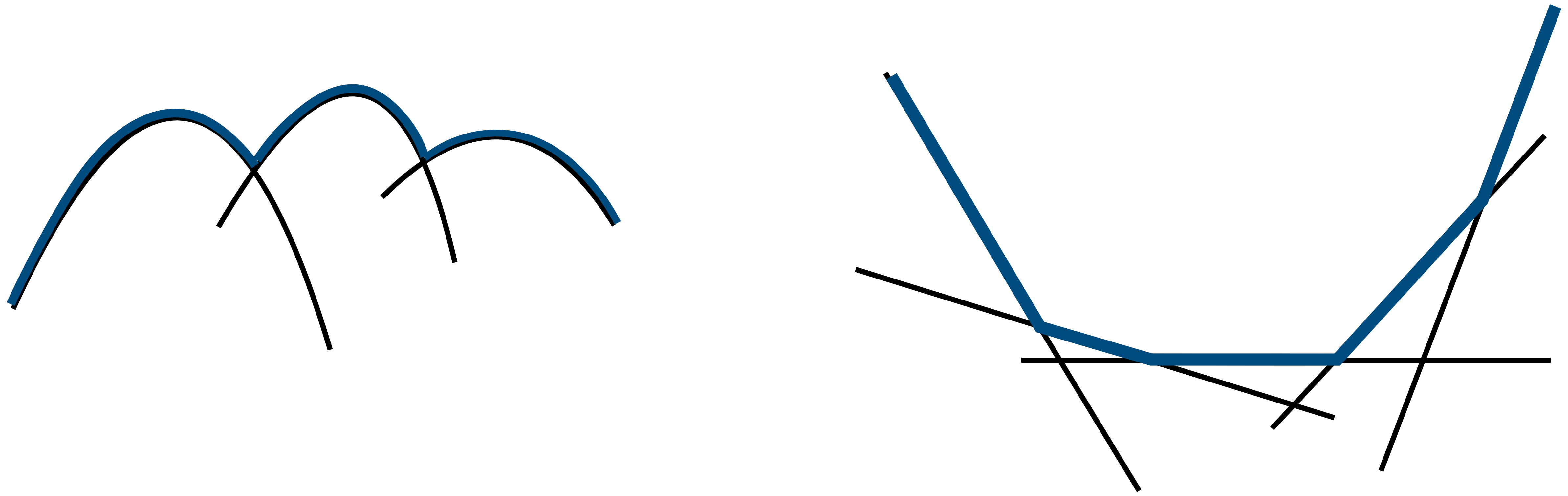

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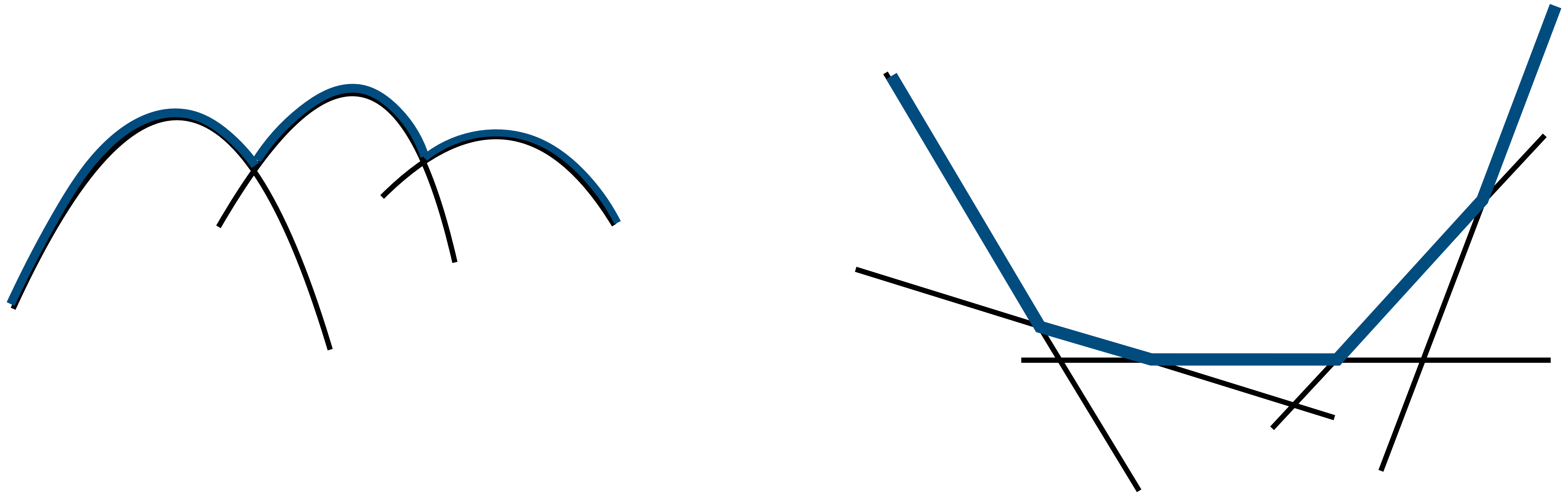

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# Worst-case values with maximum-of-concave constraints

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**Theorem** (max-of-concave function)

If each $-g_j$ is $L_j$-smooth in $u$, we have

$$\bar{g}^N(x) - \Phi^K \leq \bar{g}^K(x) \leq \bar{g}^N(x) + \frac{\max_{j \leq J} L_j}{2} D(K)$$

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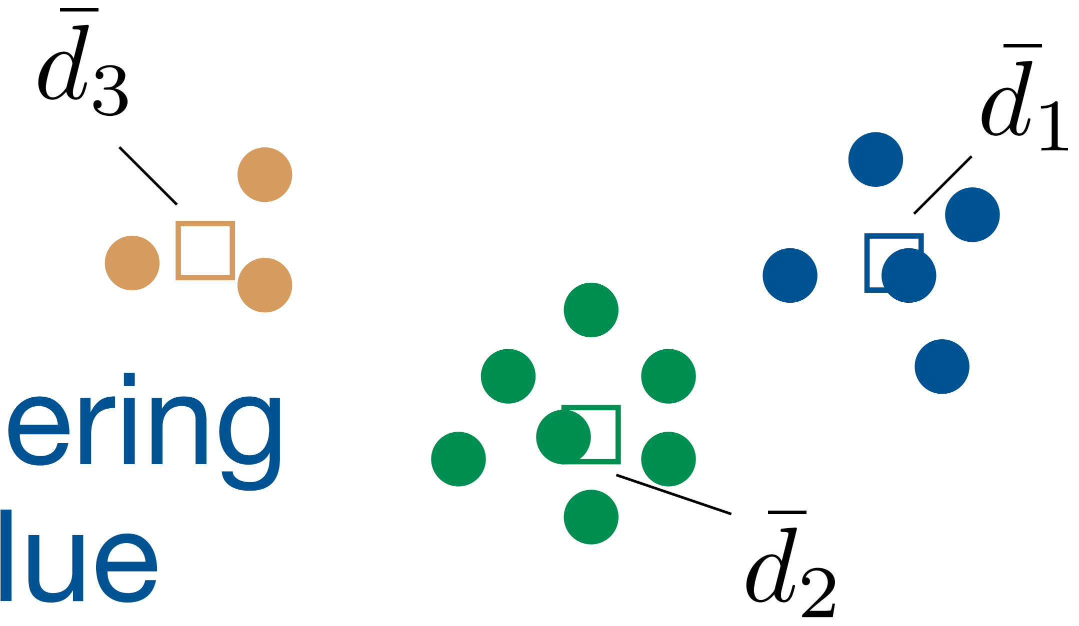

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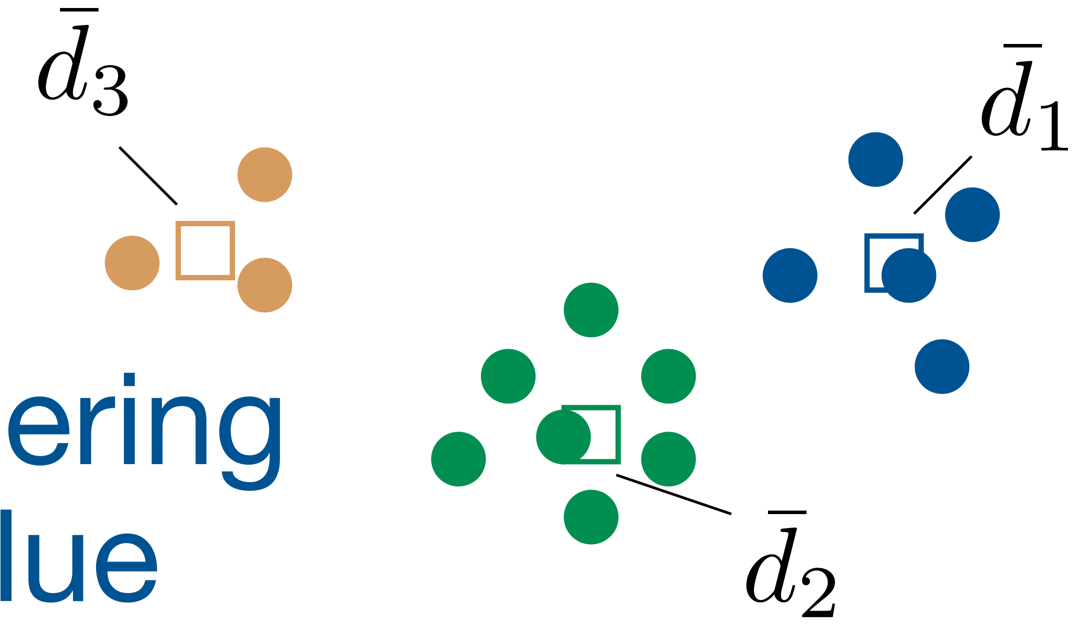
maximum of smoothness constants



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bound based on dual variables

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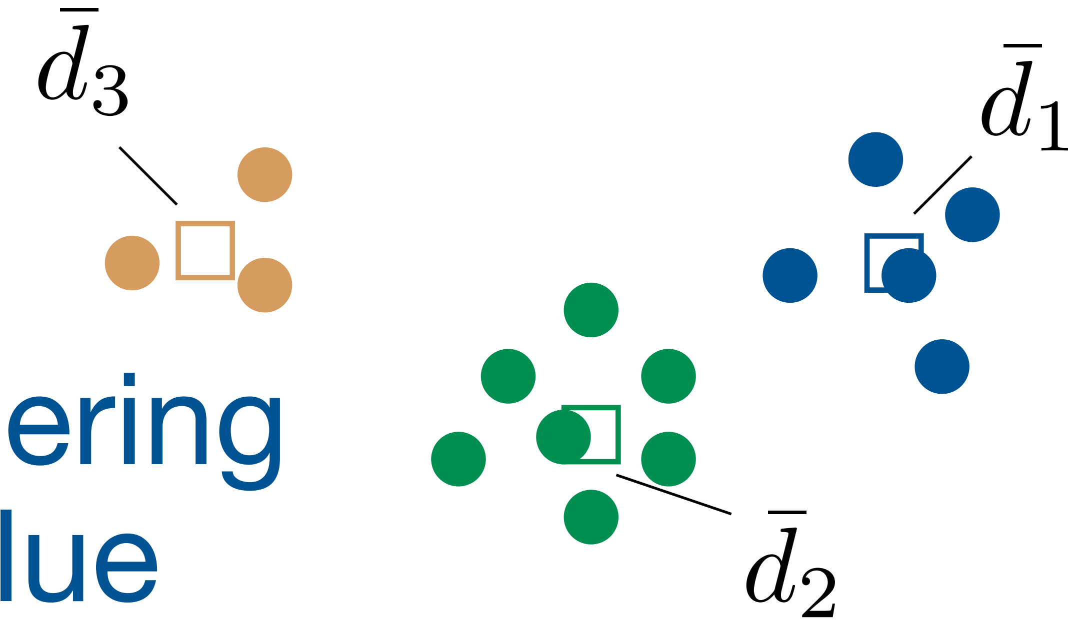

# Computational speedups on sparse portfolio optimization

$$\begin{array}{ll} \text{minimize} & \mathbf{CVaR}(-u^T x, \eta) \\ \text{subject to} & \mathbf{1}^T x = 1, \quad x \geq 0 \\ & \mathbf{card}(x) \leq C \end{array}$$

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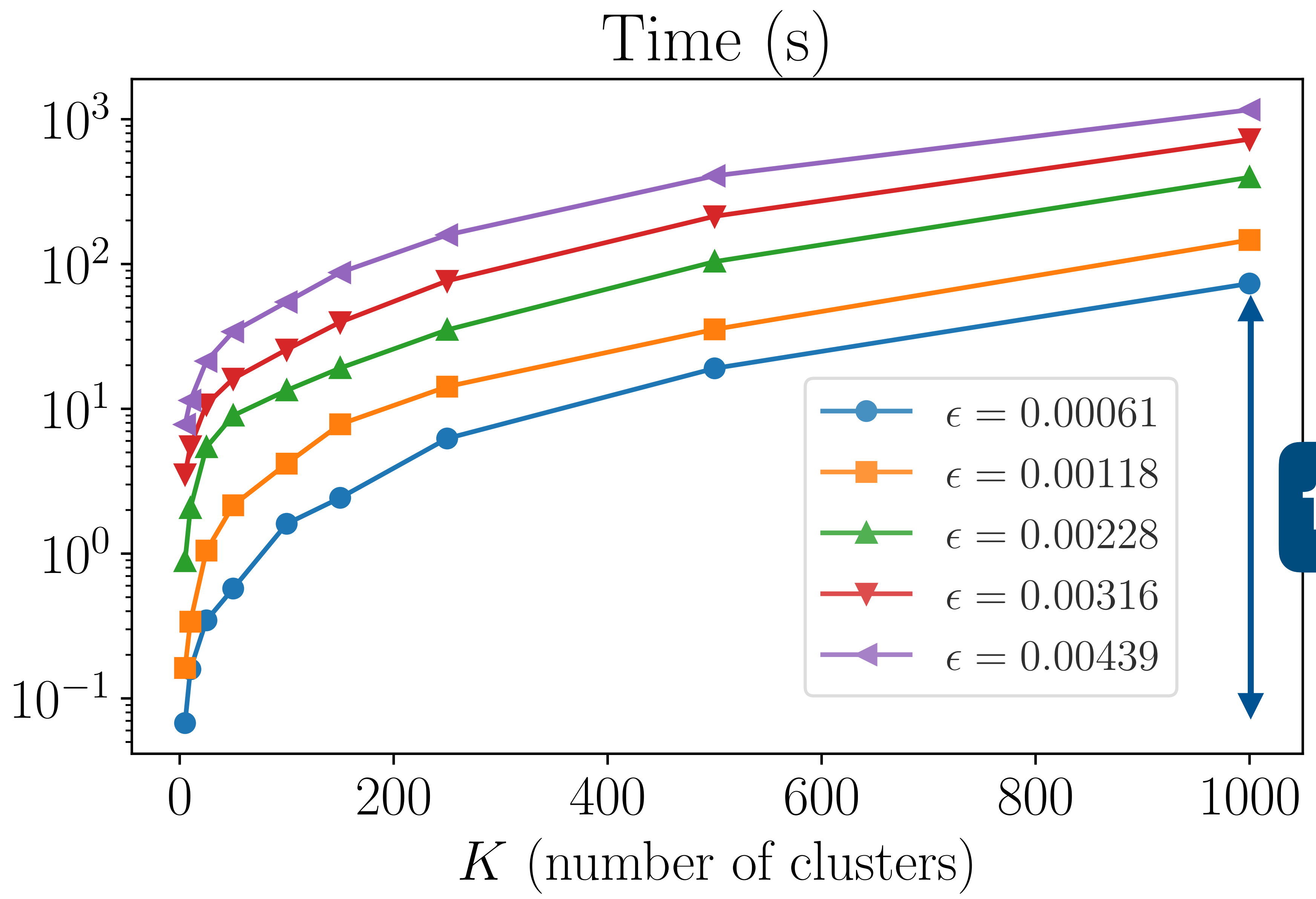

1000x

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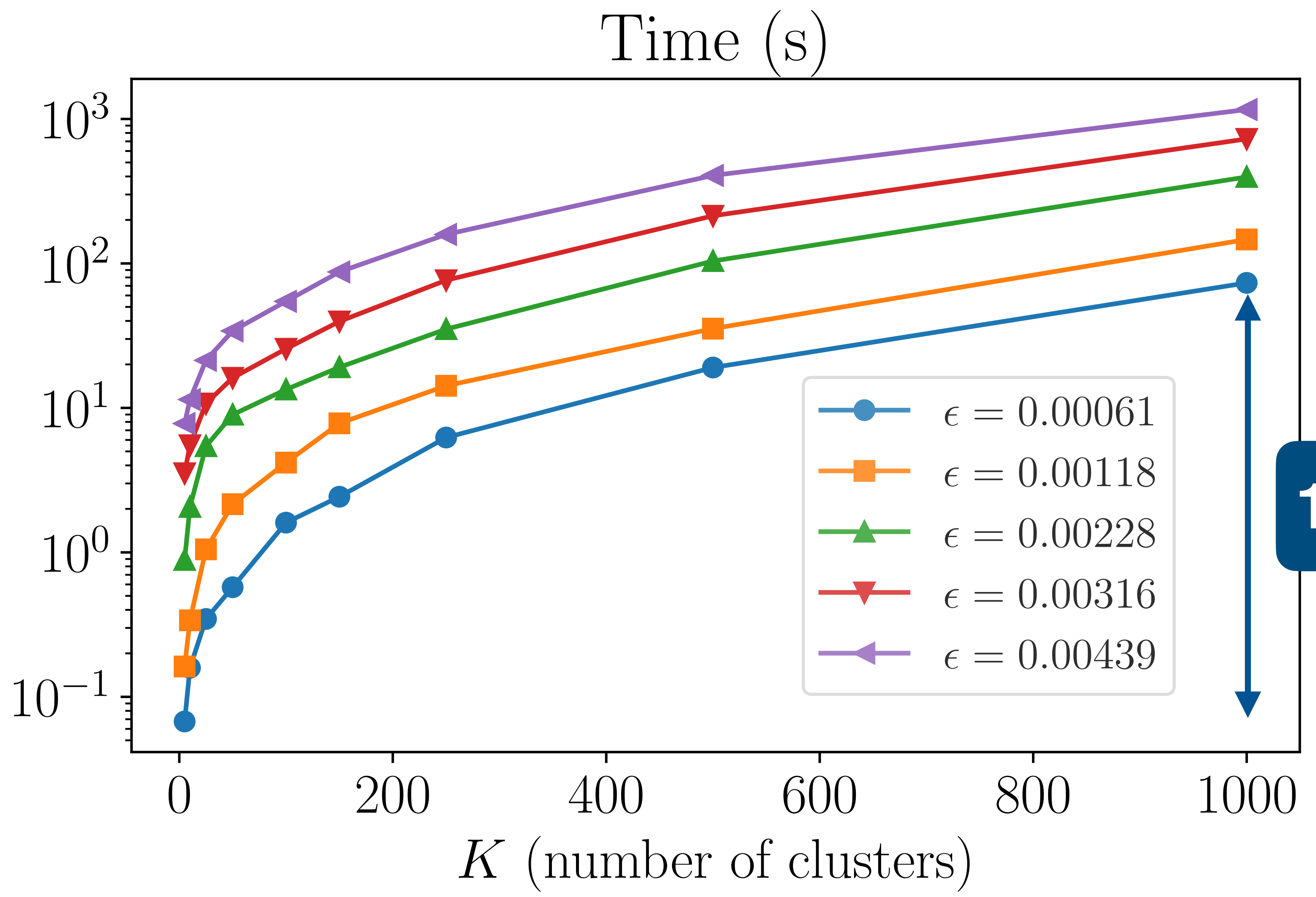

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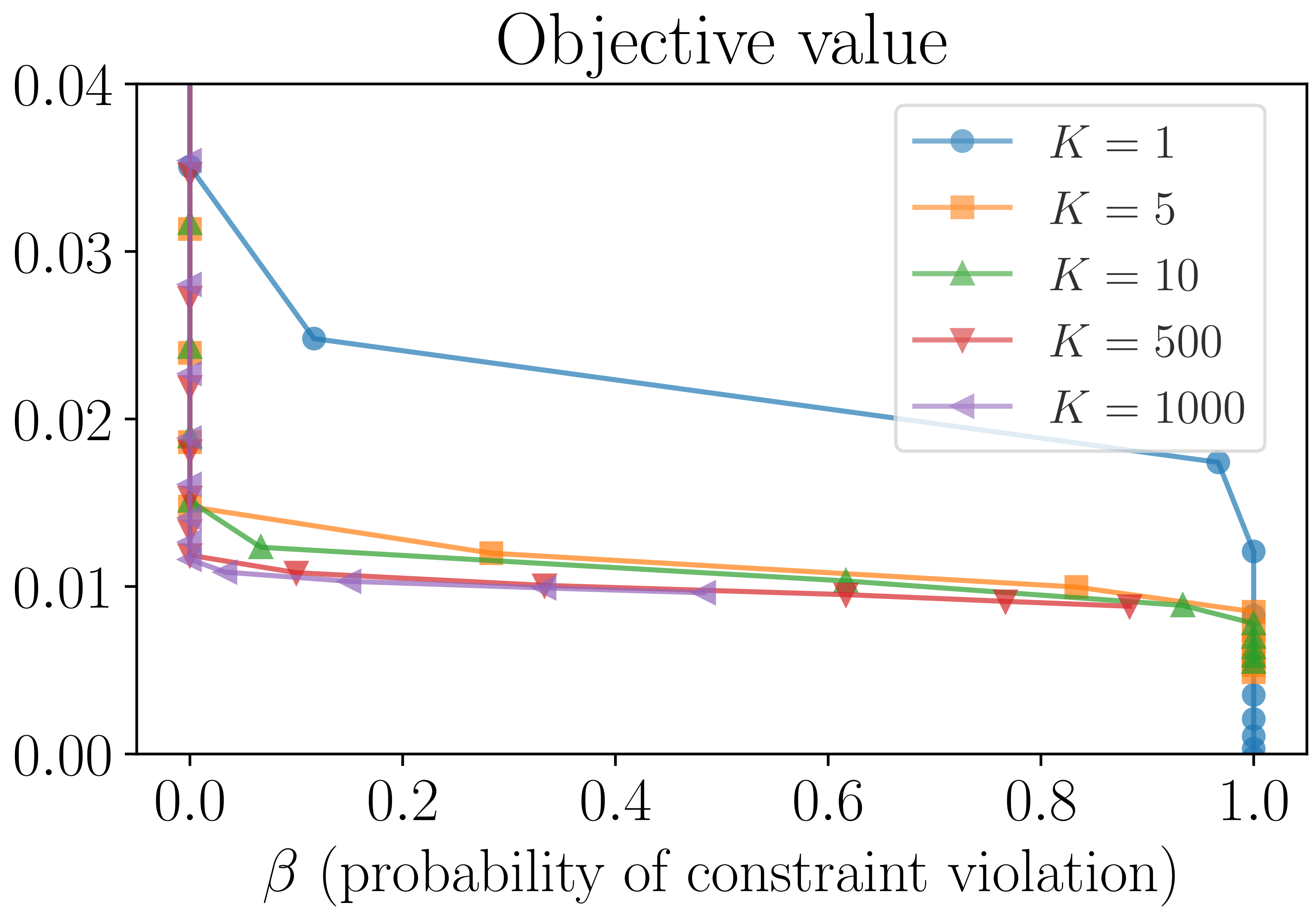

near-optimal performance with 5 clusters

# Computational speedups on capital budgeting example

$$\begin{array}{ll} \text{maximize} & \eta(u)^T x \\ \text{subject to} & a^T x \leq b \\ & x \in \{0, 1\}^n \end{array}$$

# Computational speedups on capital budgeting example

total
net present value (NPV)

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NPV of project $j$

$$\eta_j(u) = \sum_{t=1}^{T} \frac{F_{jt}}{(1+u_j)^t}$$

cash
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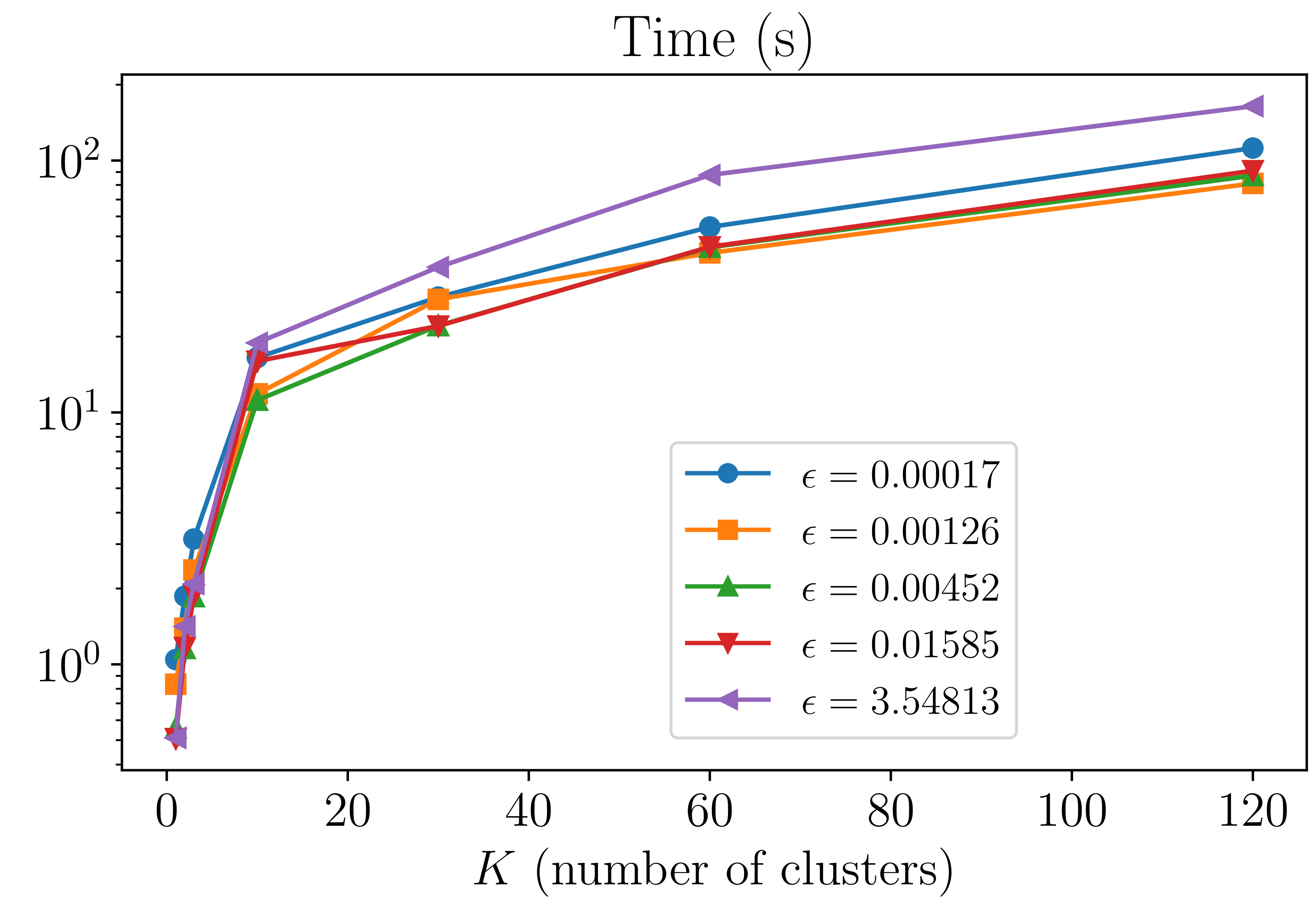

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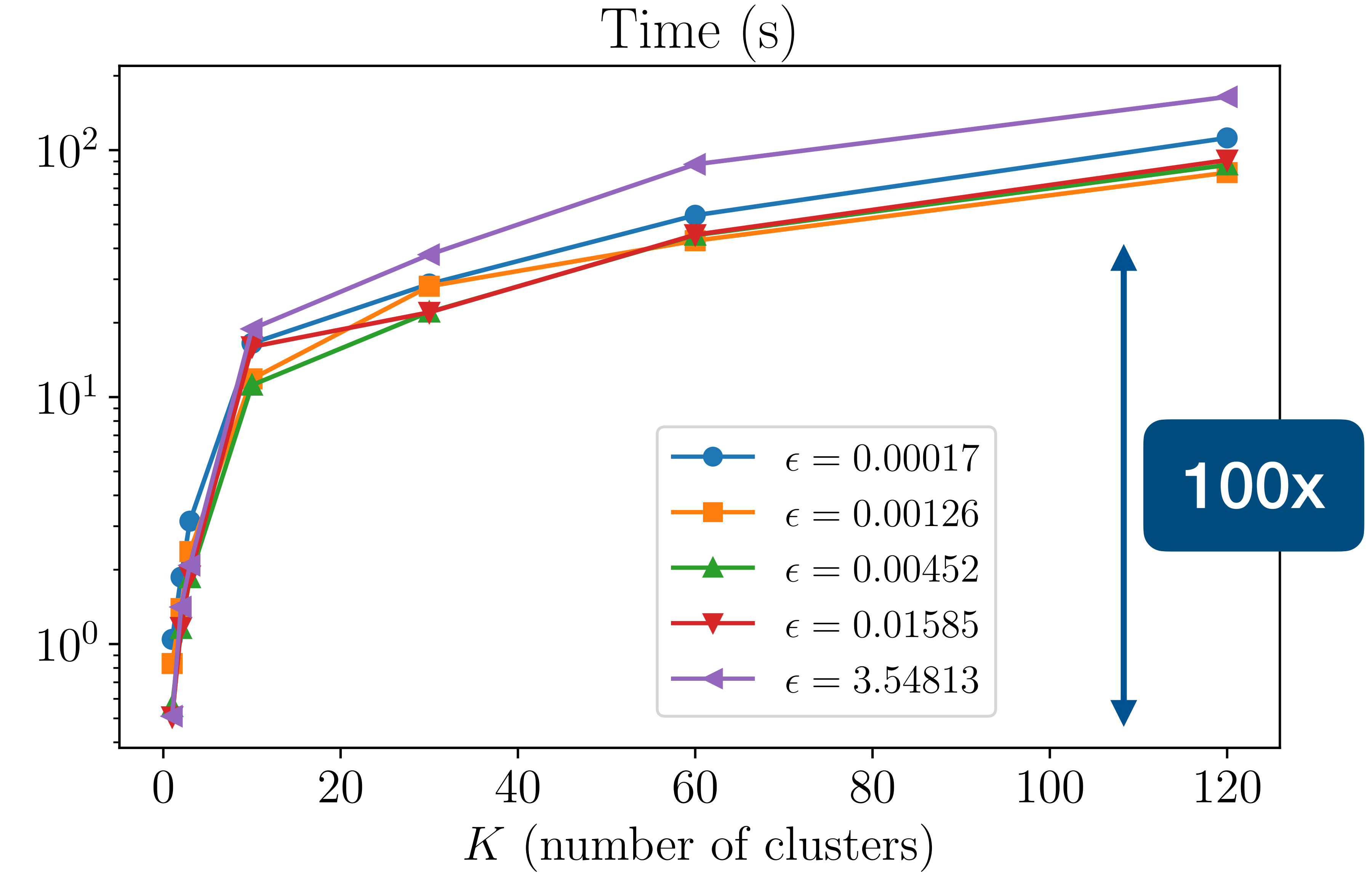

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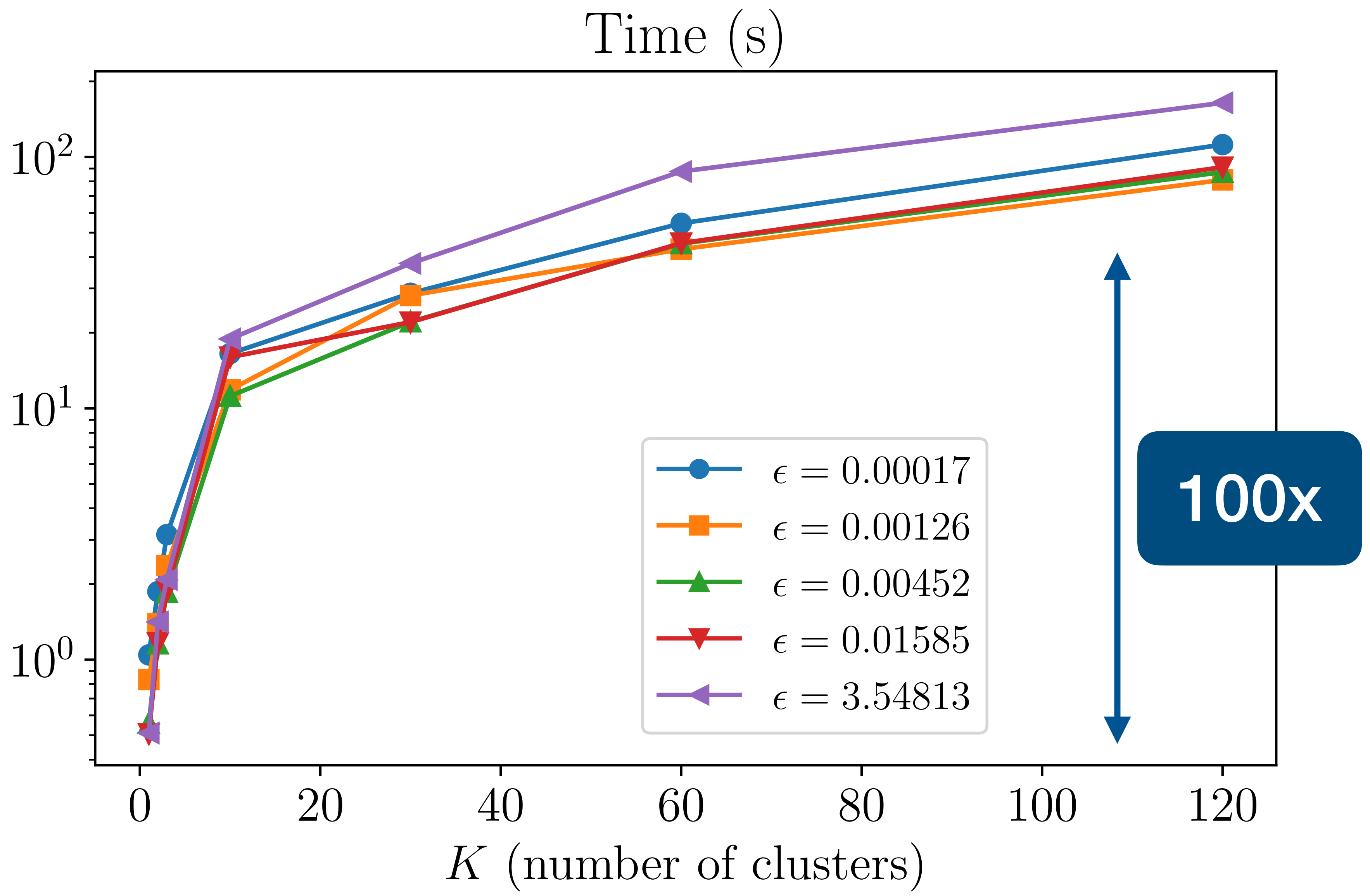
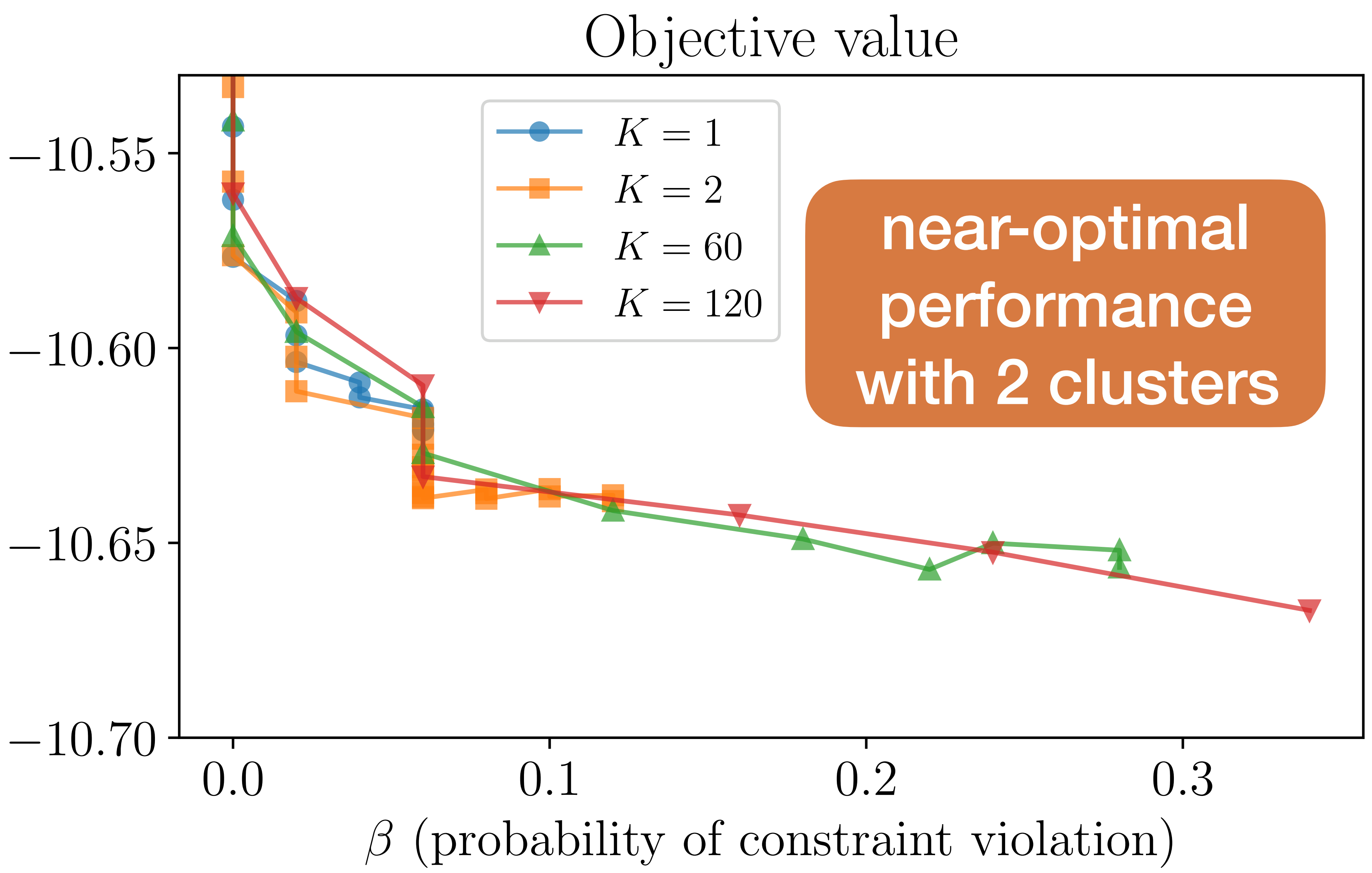


100x



near-optimal
performance
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# Mean Robust Optimization with streaming data

# Streaming data brings opportunities and challenges

streaming data

time step

$$\mathcal{D}_t = \{d_i\}_{i=1}^{t}$$

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ambiguity set

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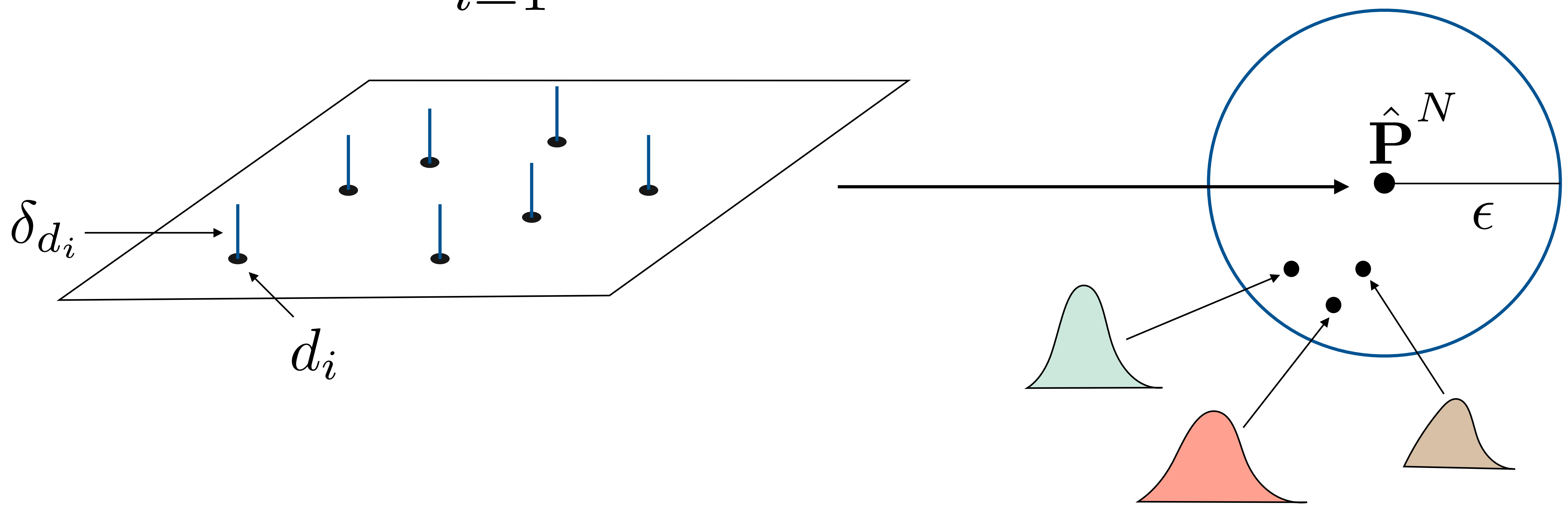

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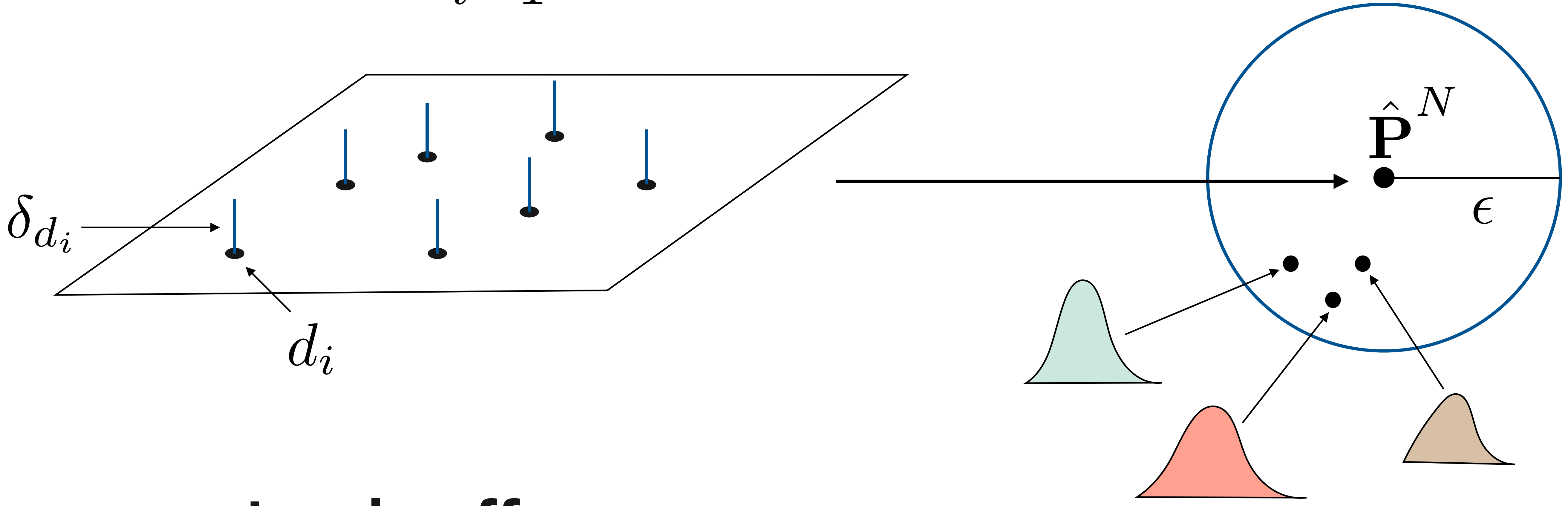

**tradeoff**

**statistics**

*shrinking radius*

$$\epsilon_t \sim t^{-1/m}$$

Fournier and Guillin (2013)

**computation**

*increasing computational cost*

$1$ datapoint $\leftrightarrow$ $1$ constraint

# Many problems with streaming data are sequential

goal

$$H_\star = \inf_{x \in X} \mathbf{E}(f(x,u))$$

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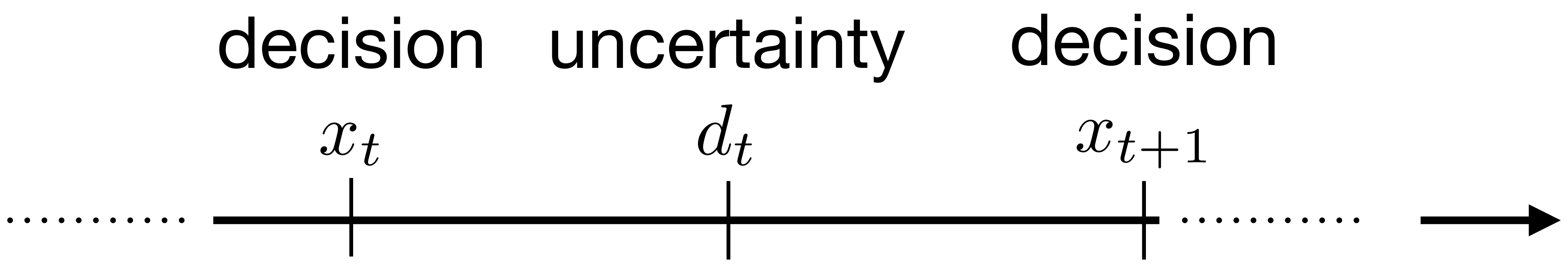

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uncertainty $d_t$

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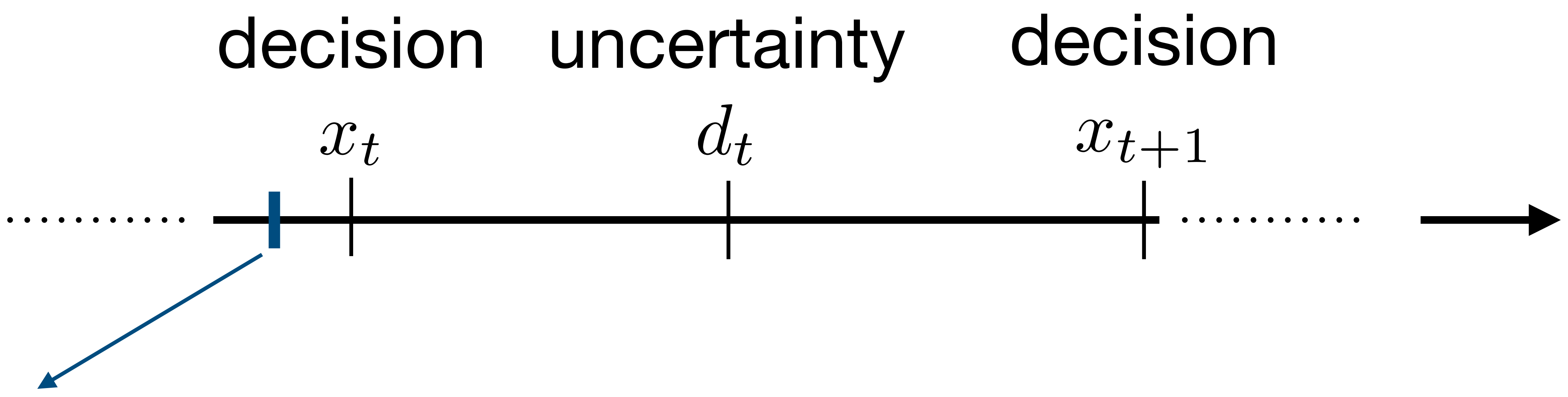

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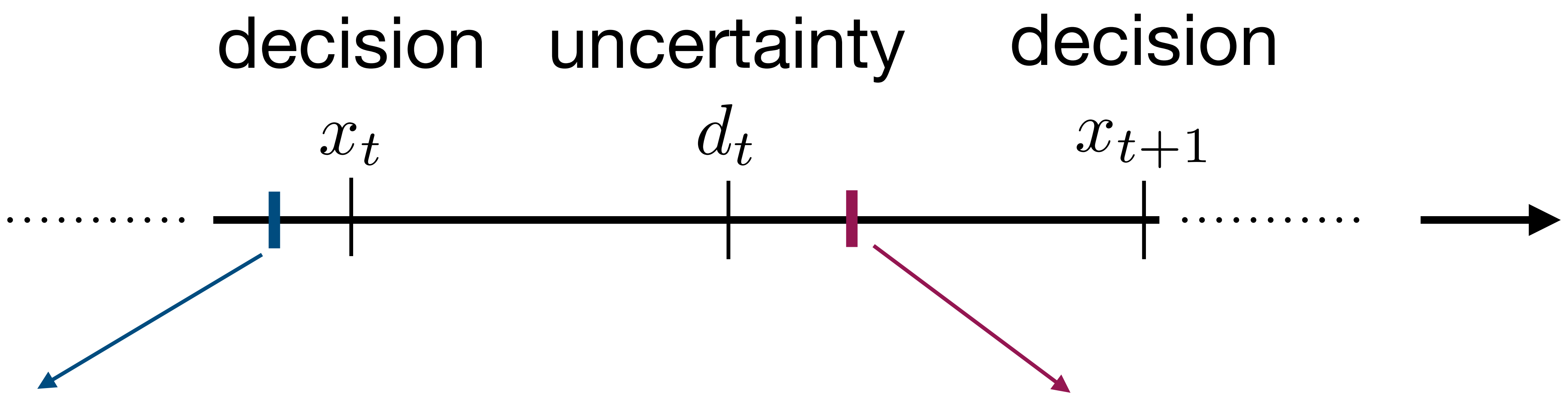

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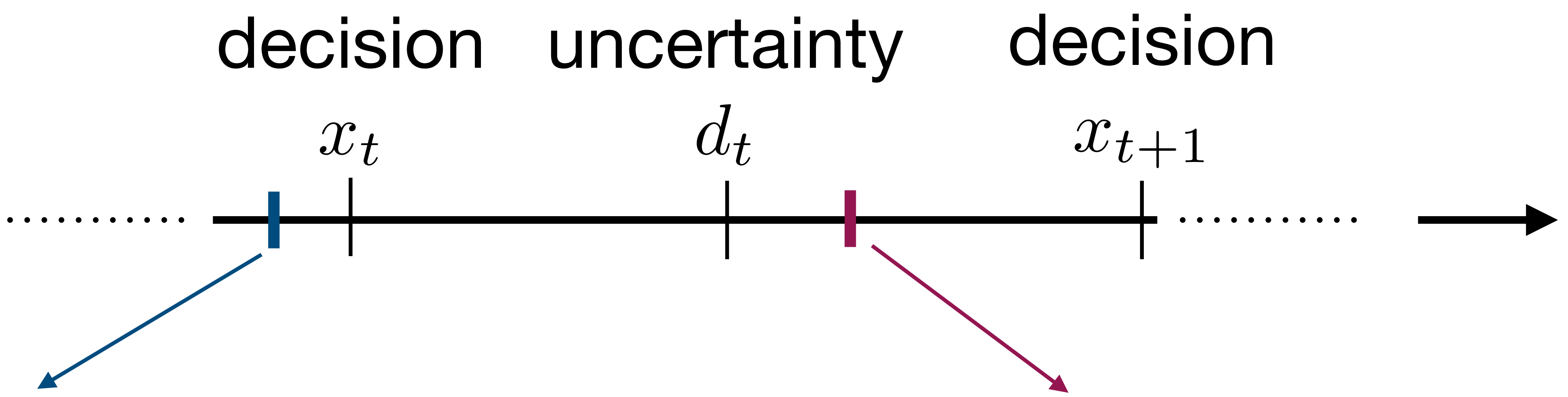

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**Can Mean Robust Optimization help?**

# Idea: compress data via online clustering

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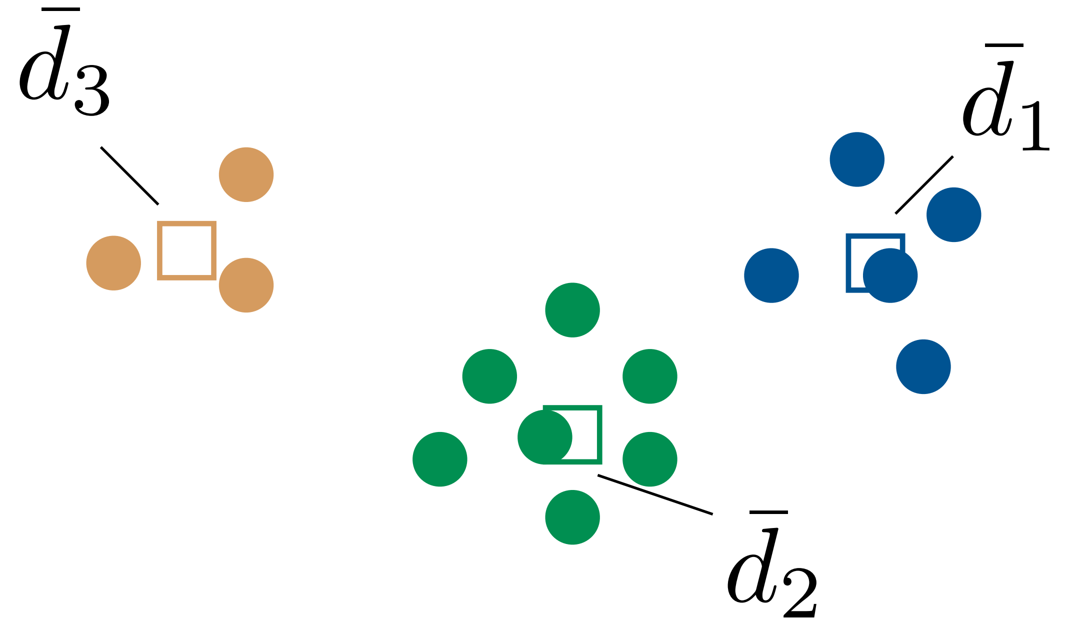

**clustered empirical distribution**

$$\hat{\mathbf{P}}_t^K = \sum_{k=1}^{K} w_t^k \delta_{\bar{d}_t^k}$$

cluster centers

cluster weights

dynamic clustering-based ambiguity set (via MRO)

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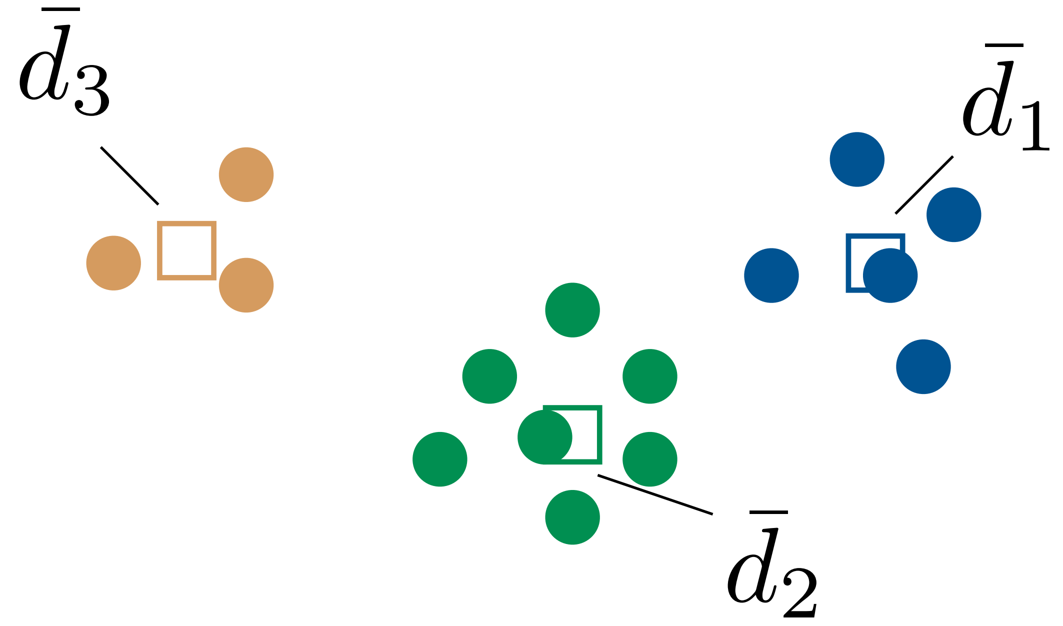

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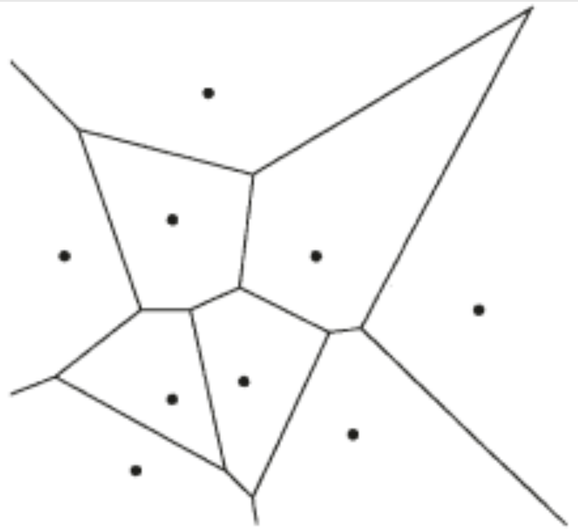
$$\longrightarrow \mathcal{P}_t^K = \left\{\mathbf{Q} \mid W_p(\hat{\mathbf{P}}_t^K, \mathbf{Q}) \le \epsilon_t\right\}$$

it works with many clustering algorithms

Fixed partitioning



Zubaroğlu and Atalay (2021)

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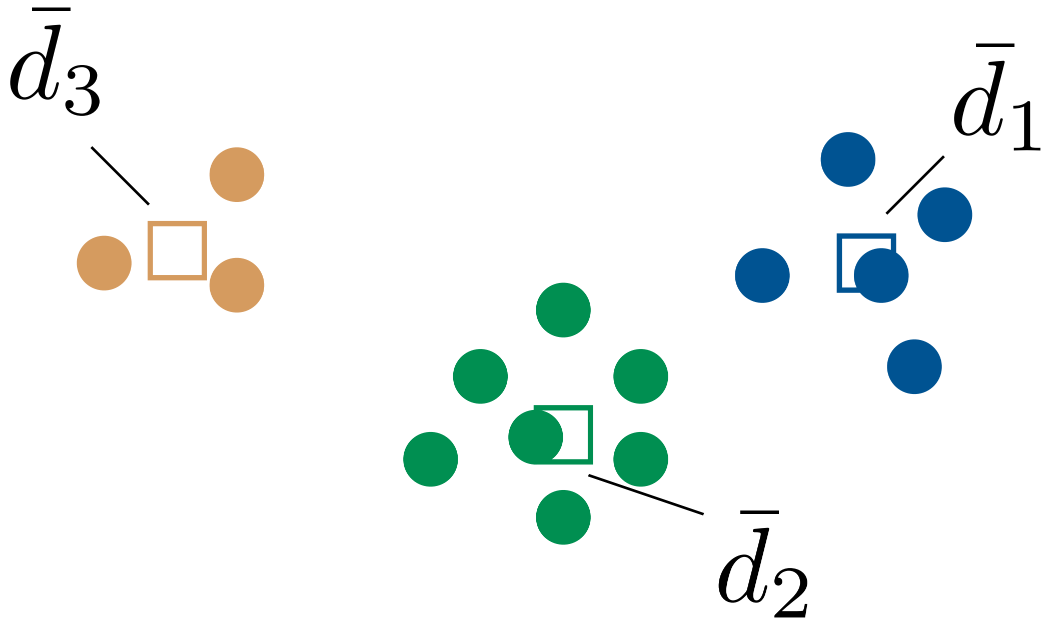

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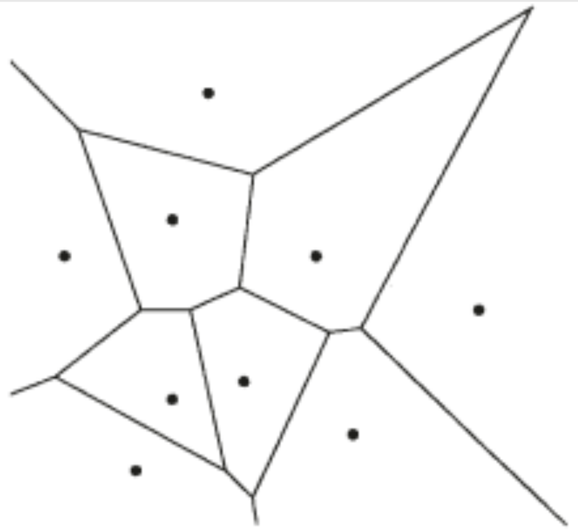

*k*-means *reclustering* at each *t*

$$\min \sum_{i=1}^{t} \sum_{k=1}^{K} a_{ik} \|d_i - \bar{d}_k\|^2$$

Zubaroğlu and Atalay (2021)

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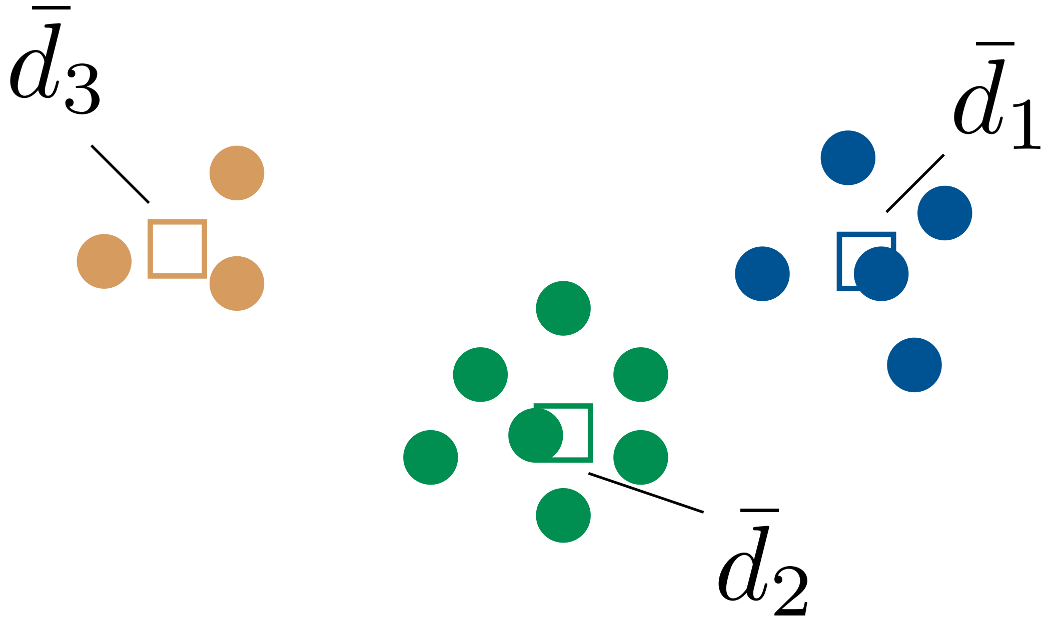

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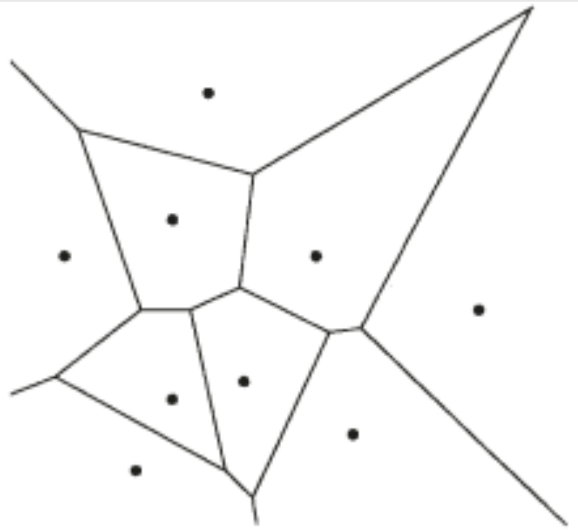

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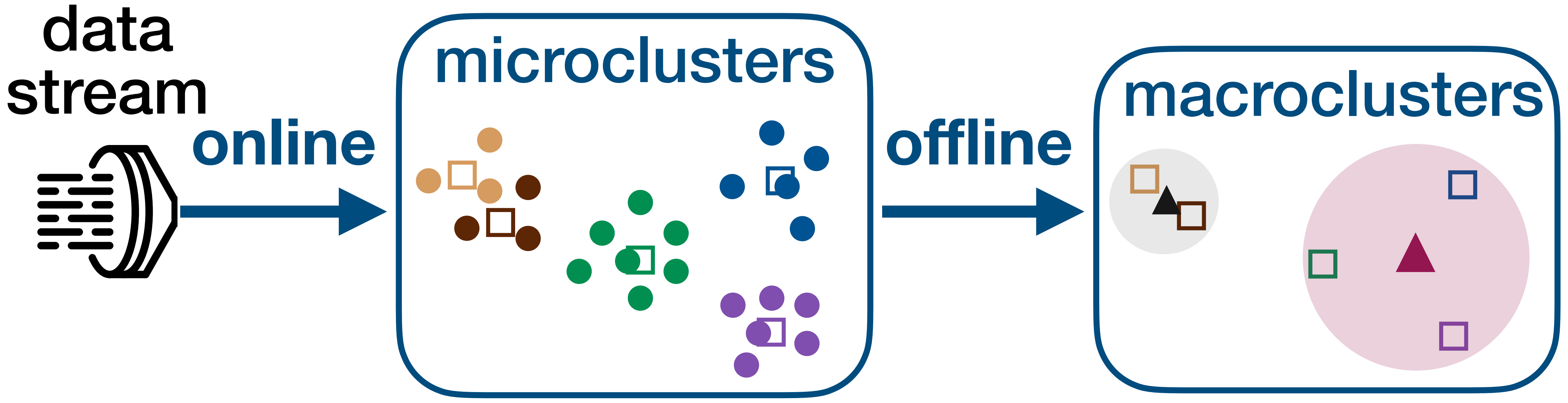
two-phase clustering



Zubaroğlu and Atalay (2021)

# Bounding the performance lost via clustering

optimal DRO value
at time $t$

$$H_t = \inf_{x \in X} \sup_{\mathbf{P} \in \mathcal{P}_t} \mathbf{E}(f(x,u))$$

optimal MRO value
at time $t$ with $K$ clusters

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# Bounding the performance lost via clustering

optimal DRO value
at time $t$

$$H_t = \inf_{x \in X} \sup_{\mathbf{P} \in \mathcal{P}_t} \mathbf{E}(f(x,u))$$

optimal MRO value
at time $t$ with $K$ clusters

$$H_t^K = \inf_{x \in X} \sup_{\mathbf{P} \in \mathcal{P}_t^K} \mathbf{E}(f(x,u))$$

### Theorem (concave function)

If $f$ is $L$-smooth and $M$-Lipschitz in $u$, we have

$$H_t \leq H_t^K \leq H_t + \min\left\{\frac{L}{2} D_t(K), M(2\epsilon_t + W_p(\hat{P}_t, \hat{P}_t^K))\right\}$$

# Bounding the performance lost via clustering

optimal DRO value at time $t$

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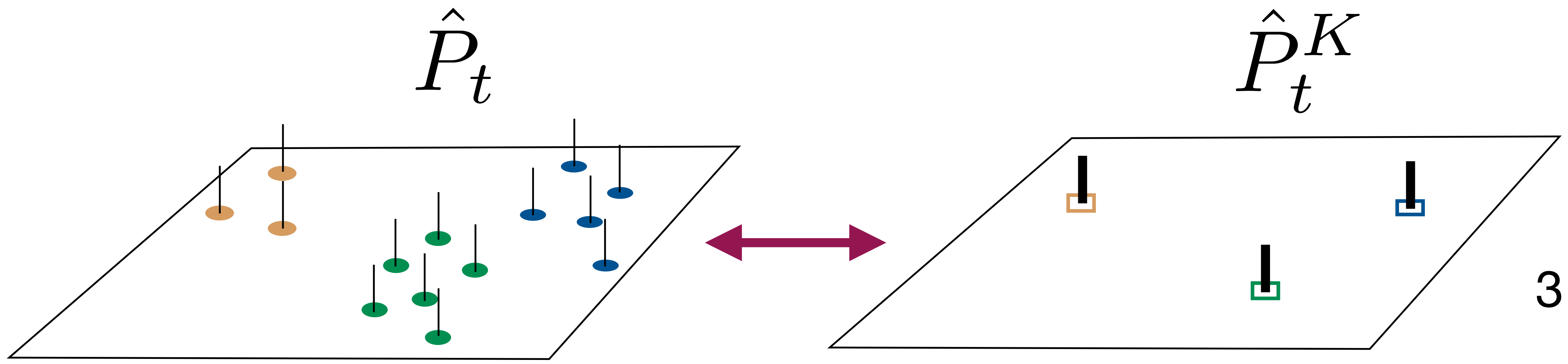
Wasserstein distance



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value at time $t$

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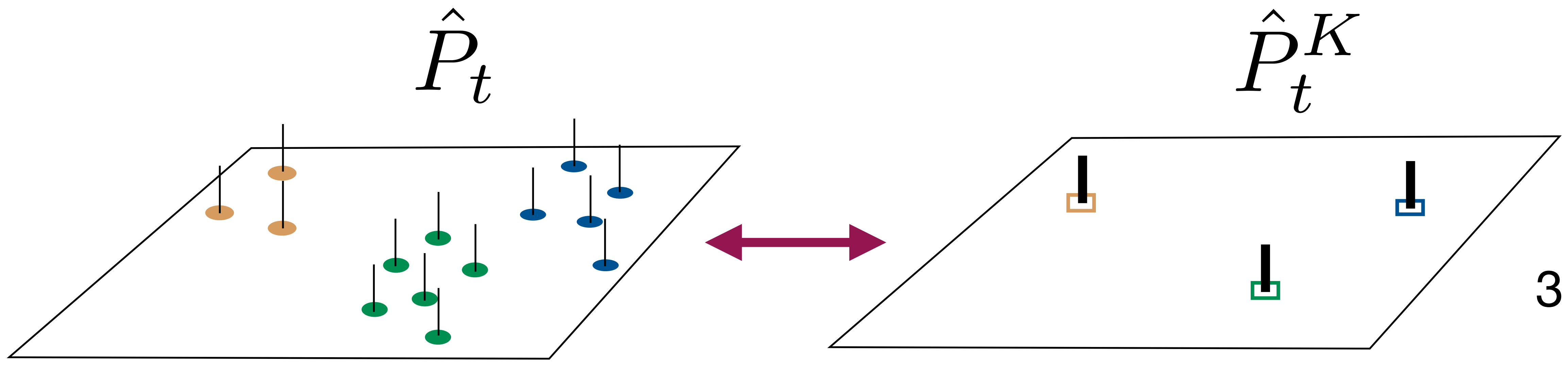
radius
*same as DRO*
Gao (2022)

$$\tilde{O}(1/\sqrt{t})$$

Wasserstein distance



# Computational speedups on online sparse portfolio with high confidence

$$\begin{array}{ll} \text{minimize} & \mathbf{CVaR}(-u^T x, \eta) \\ \text{subject to} & \mathbf{1}^T x = 1, \quad x \geq 0 \\ & \mathbf{card}(x) \leq C \end{array}$$

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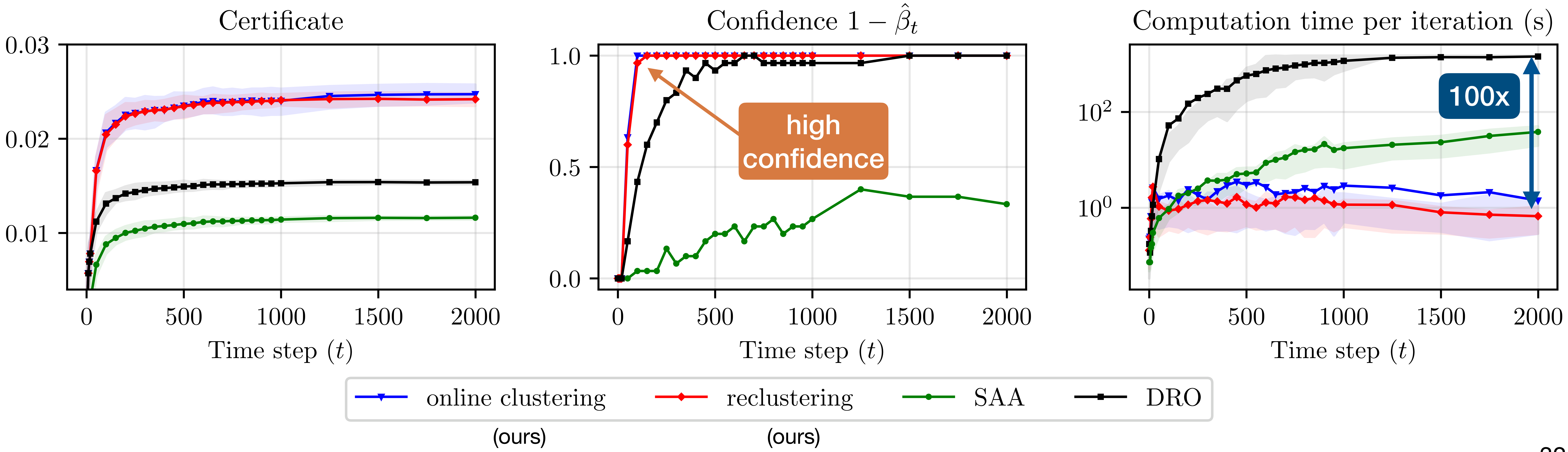

# Mean Robust Optimization

## Using data wisely in decision-making under uncertainty

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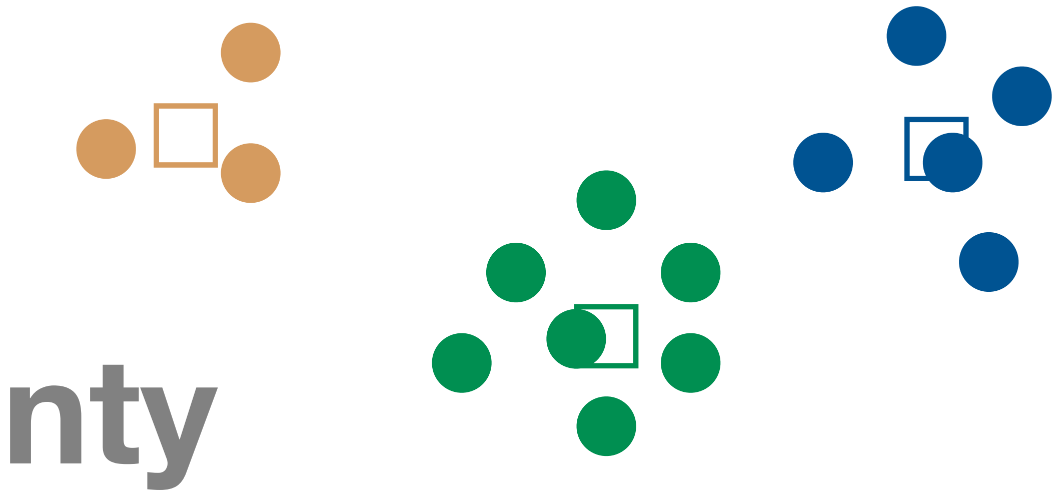
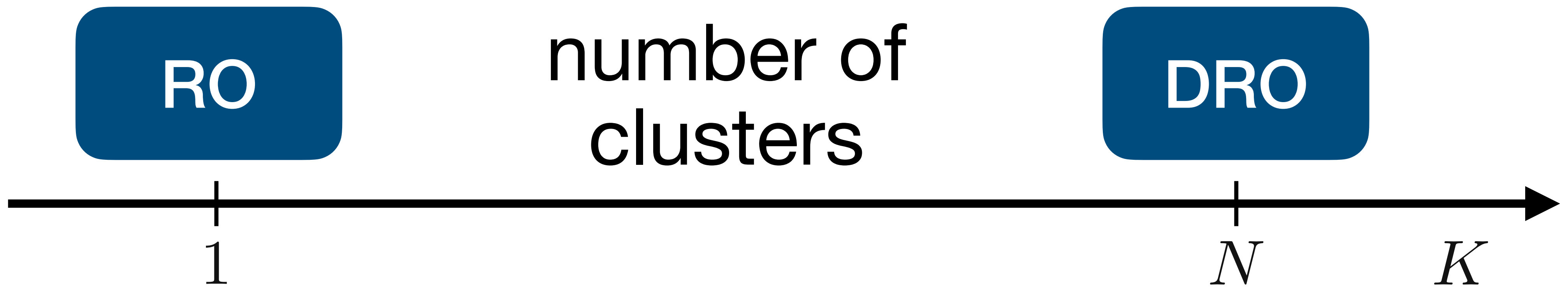


- **Bridges** RO and DRO



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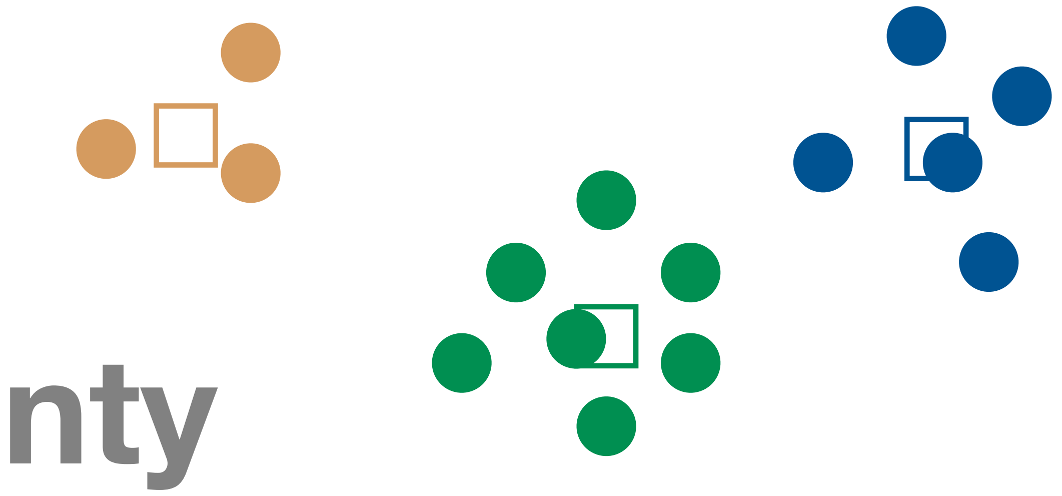

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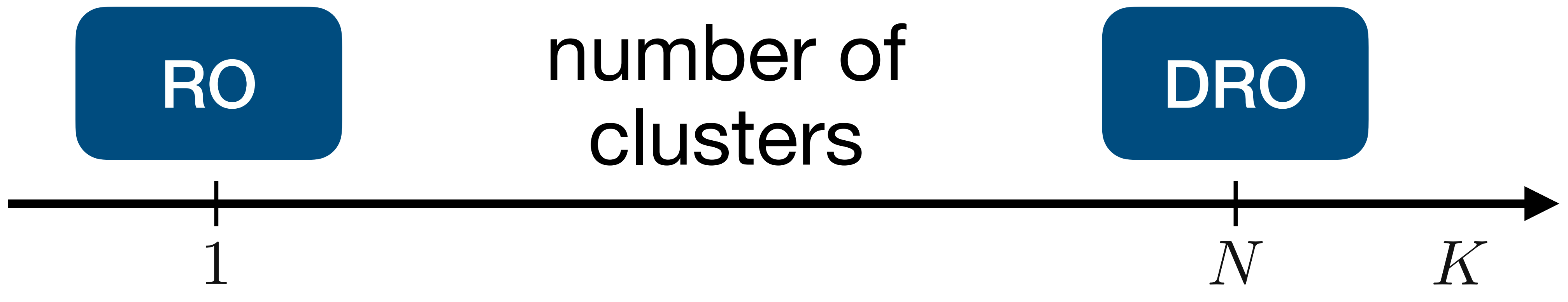

- Clustering effect
  - $g$ affine in $u$ → **zero clustering effect!**
  - $g$ (max of) concave in $u$ → **performance bound**

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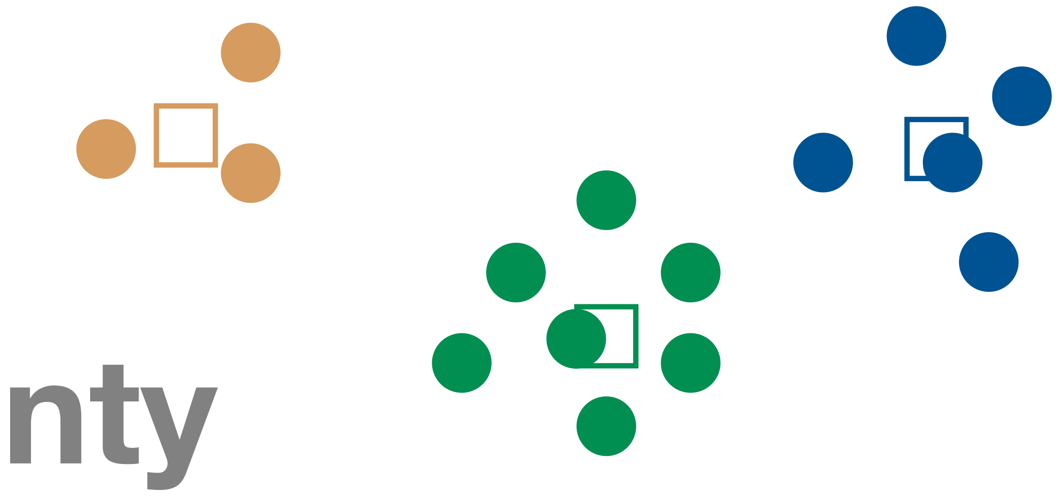

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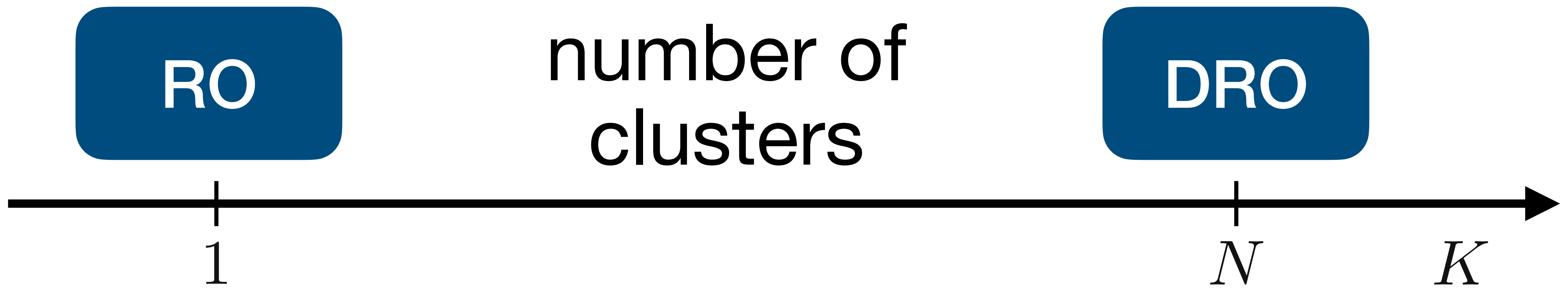

RO

number of clusters

DRO

$1$ $N$ $K$

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- Multiple orders of magnitude speedups

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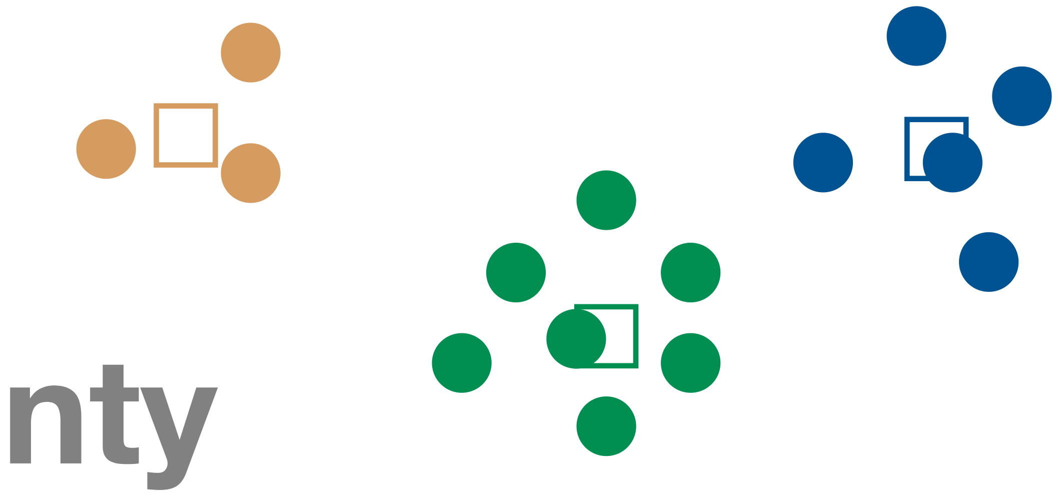

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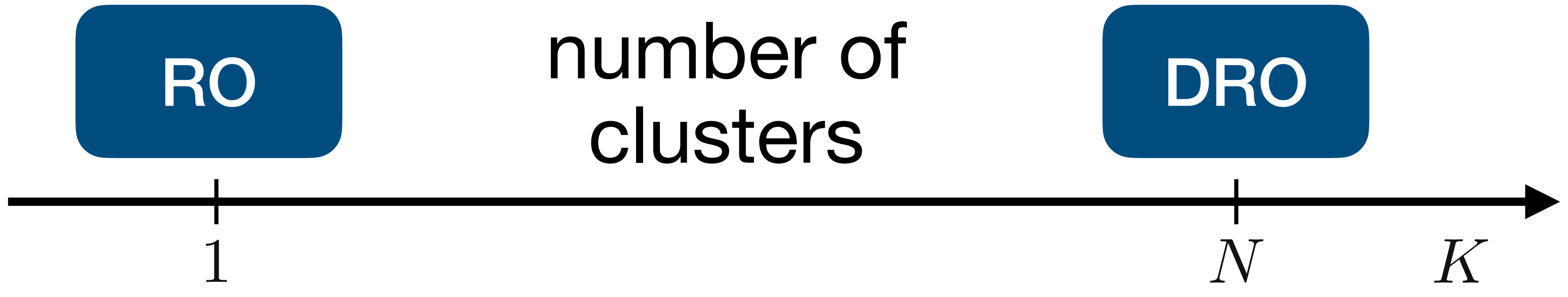

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- Works well with **streaming data**

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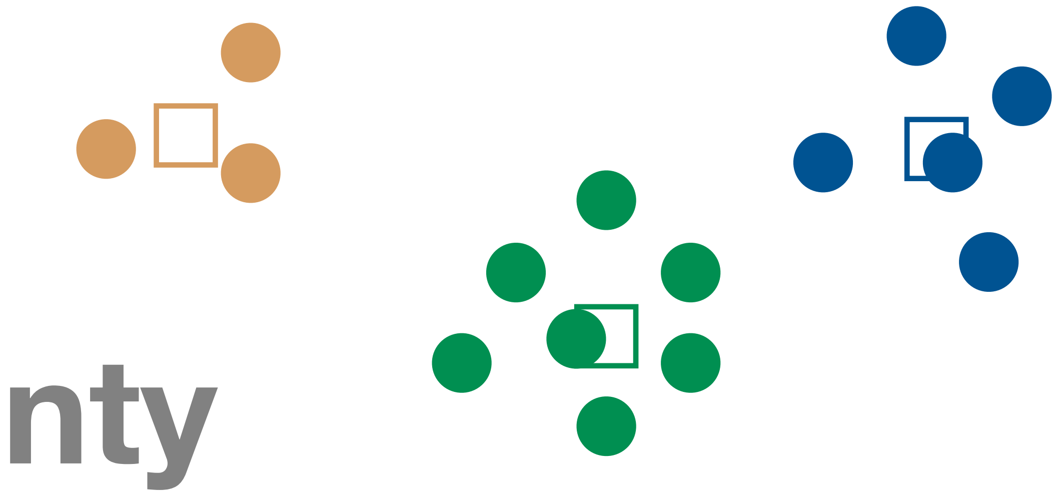

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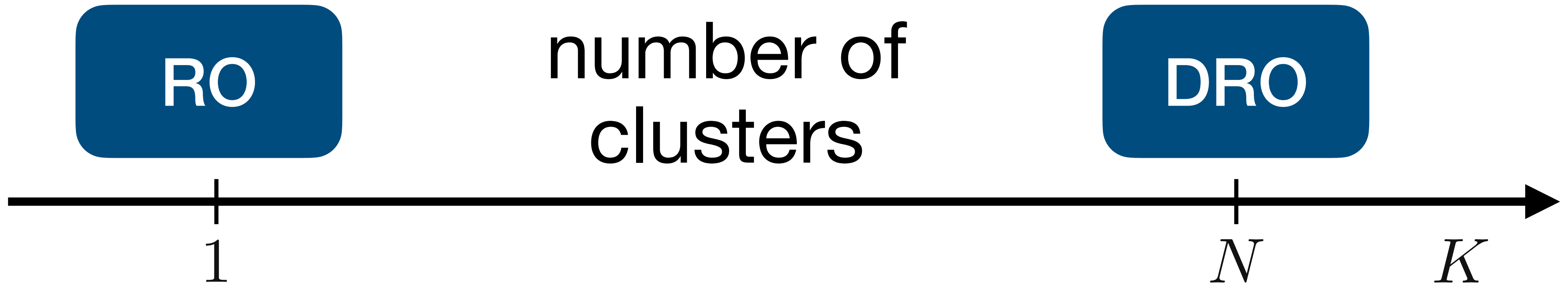

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**Mean Robust Optimization**
I. Wang, C. Becker, B. Van Parys, and B. Stellato
*Mathematical Programming*, 2024
github.com/stellatogrp/mro_experiments

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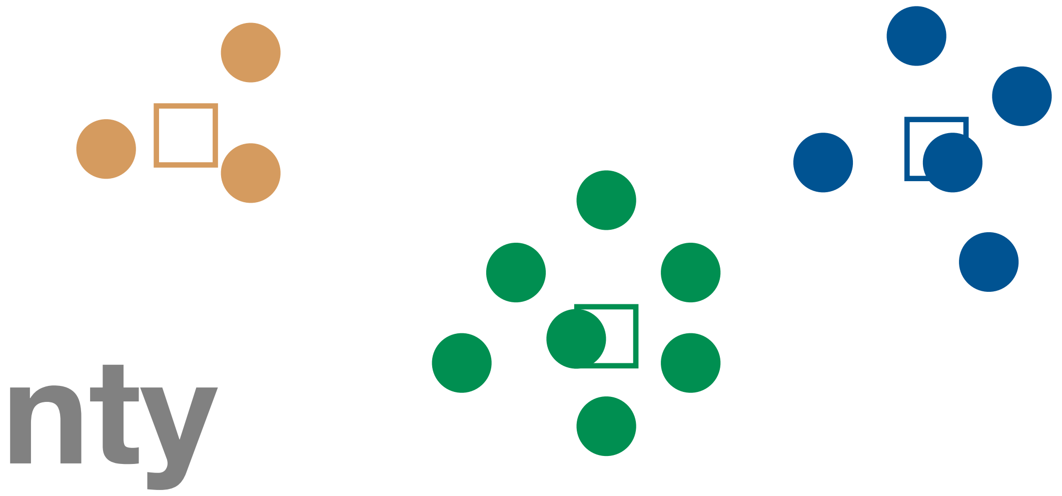

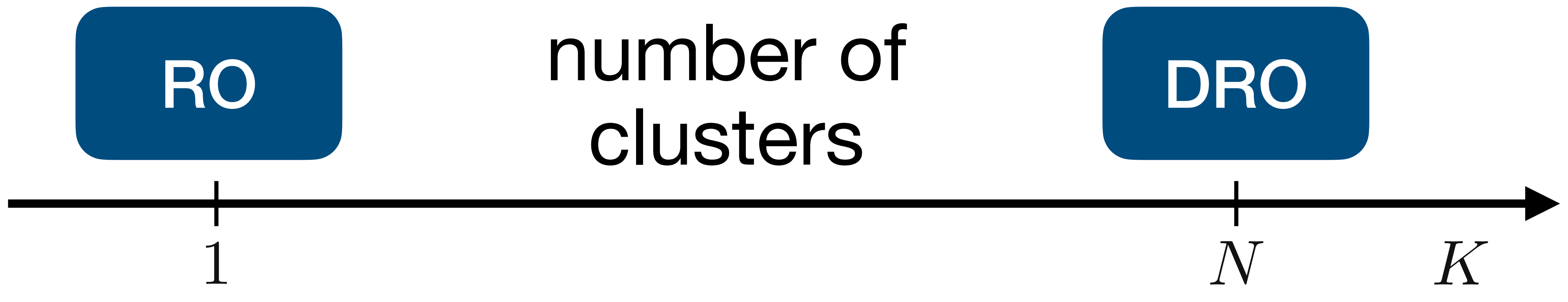

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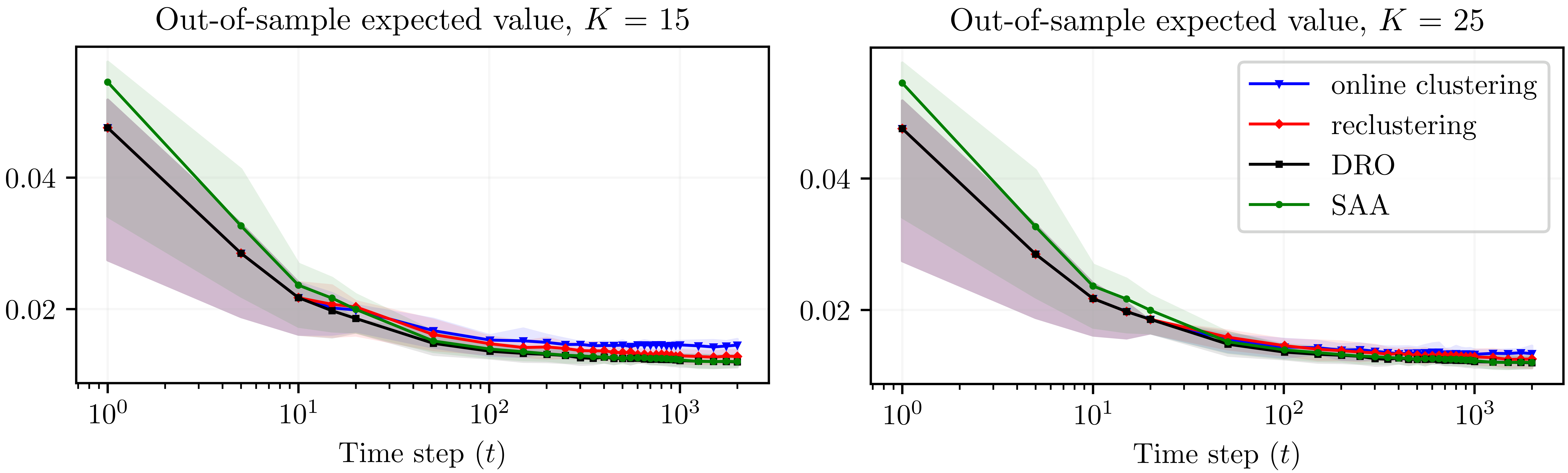
# Backup

# Online out-of-sample performance


Out-of-sample expected value, $K = 15$

Out-of-sample expected value, $K = 25$

**Obtain near-optimal out-of-sample values**