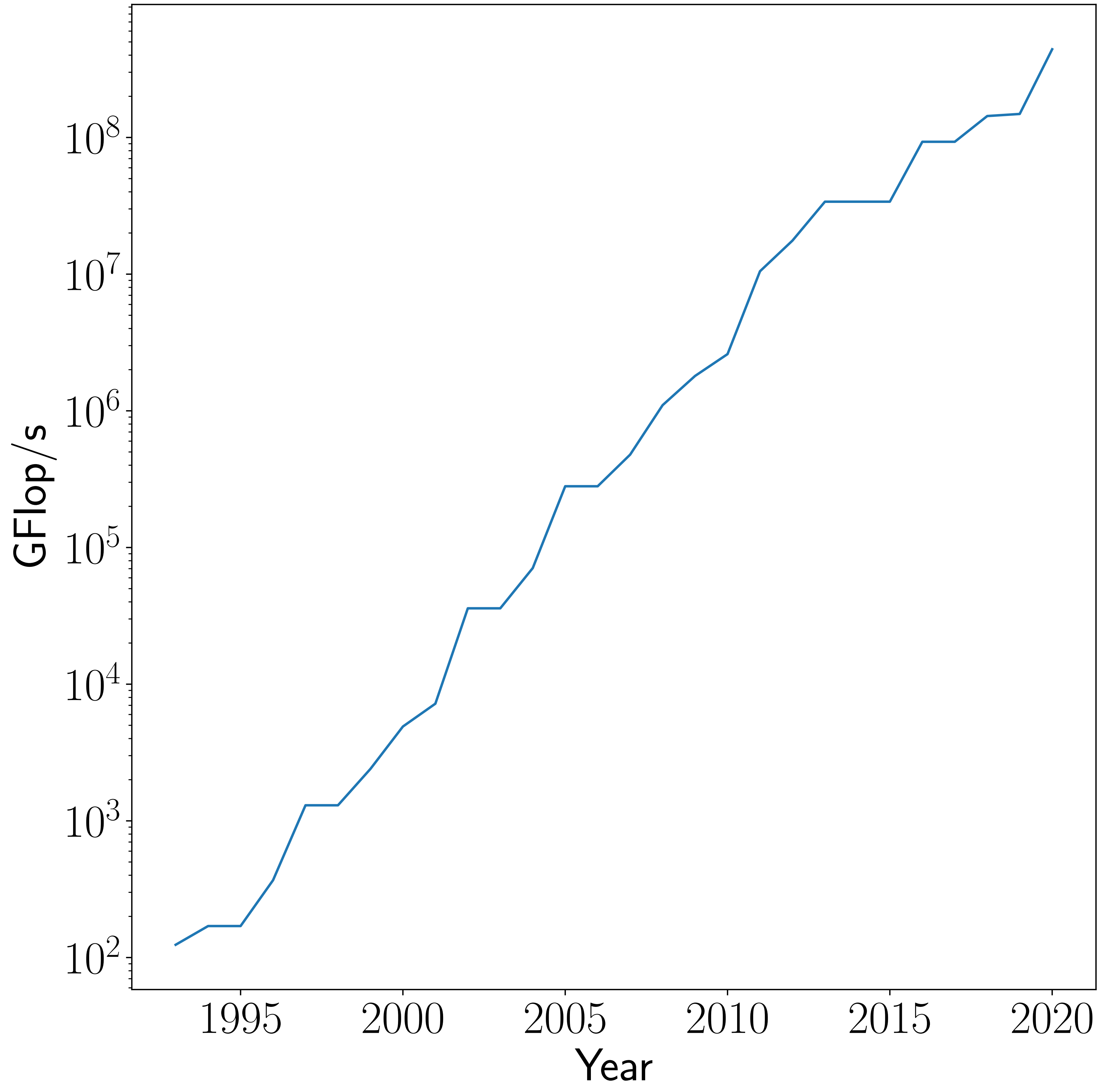
Data-Driven Embedded Optimization for Control

PRINCETON UNIVERSITY ORFE

Bartolomeo Stellato — Multi-Agent Tech Talks, NASA JPL, March 8 2022

Tremendous progress in optimization

Top500 peak CPU power



Hardware + Software

400 billion times speedups!

400,000 years → 30 seconds

[Our World in Data, https://github.com/owid/owid-datasets]

Is it enough?

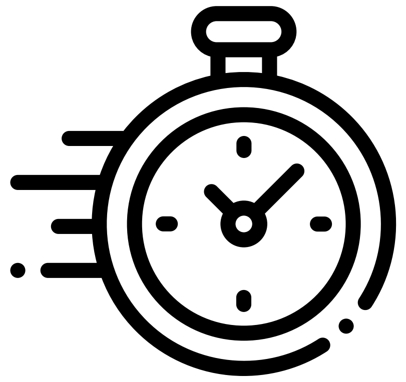
400,000 years → 30 seconds

Is it enough?

400,000 years → 30 seconds

Robotics

< 10 milliseconds



Is it enough?

400,000 years → 30 seconds

Robotics

< 10 milliseconds

High-Frequency Trading

< 1 millisecond

Same problem with varying data

$$\begin{array}{ll} \text{minimize} & f(x, \theta) \\ \text{subject to} & g(x, \theta) \leq 0 \end{array}$$

Same problem with varying data

$$\begin{array}{ll} \text{minimize} & f(x, \theta) \\ \text{subject to} & g(x, \theta) \leq 0 \end{array}$$

decisions

Same problem with varying data

data

$$\begin{array}{ll} \text{minimize} & f(x, \theta) \\ \text{subject to} & g(x, \theta) \leq 0 \end{array}$$

decisions

Same problem with varying data

data

$$\begin{array}{ll} \text{minimize} & f(x, \theta) \\ \text{subject to} & g(x, \theta) \leq 0 \end{array}$$

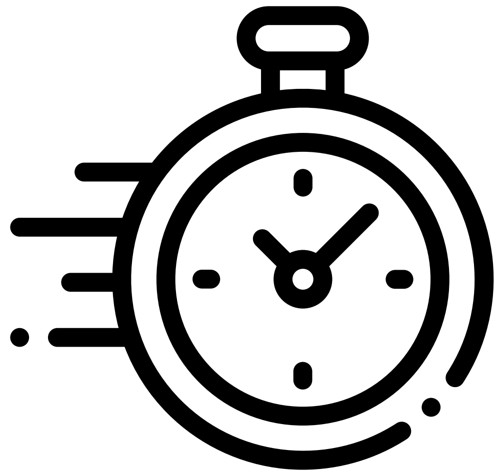
decisions

Can we solve it in **milliseconds** *or* **microseconds**?

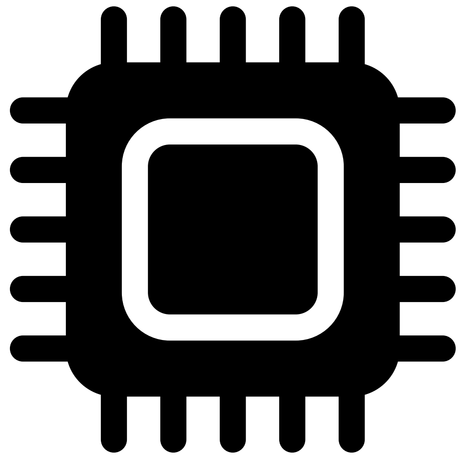
Challenges in real-time optimization

Hardware

Real-Time



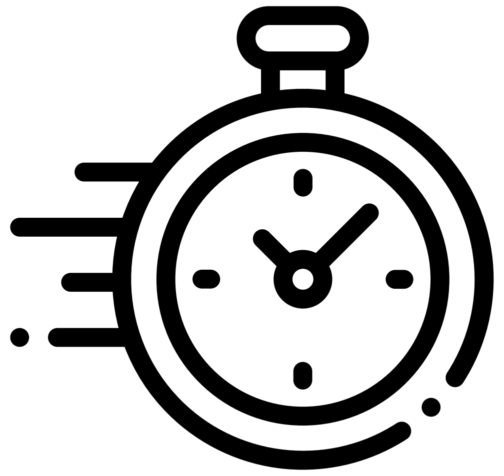
Limited resources



Challenges in real-time optimization

Hardware

Real-Time



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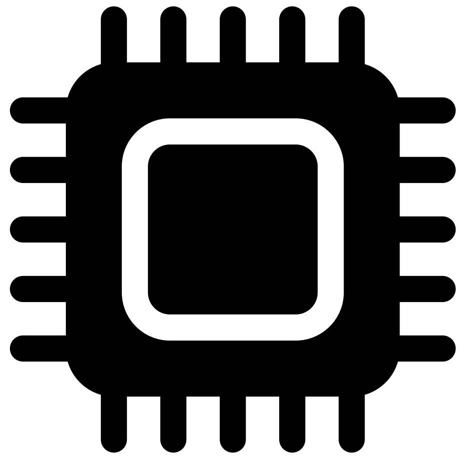
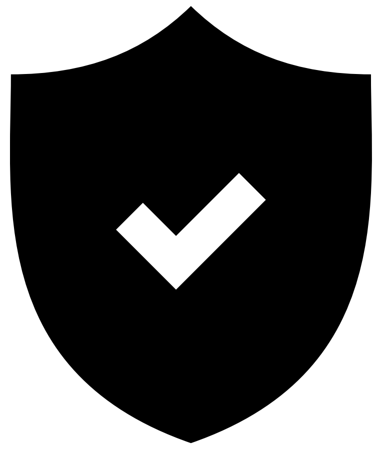
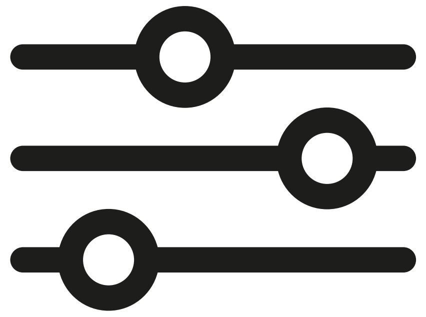


Software

Reliability



Easy
tuning

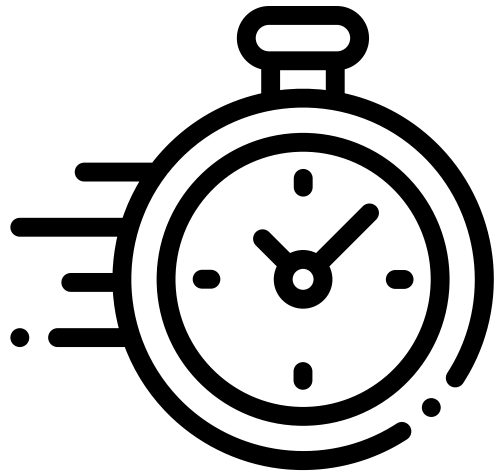


Today's talk

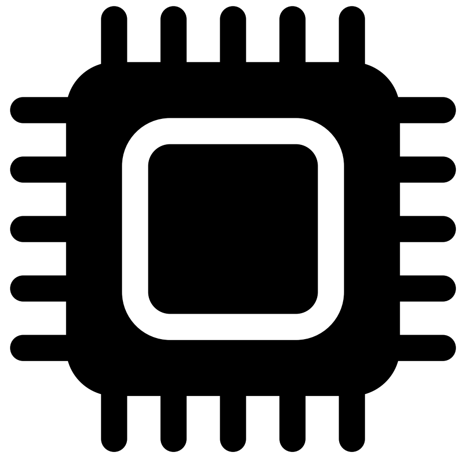
Data-Driven Embedded Optimization for Control

OSQP Solver

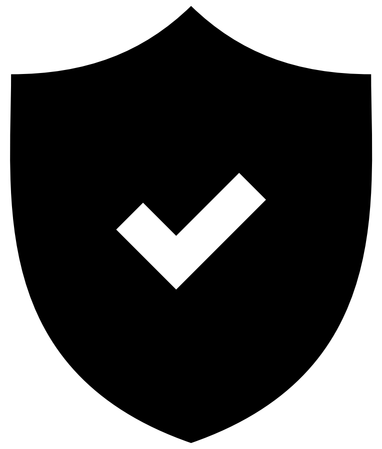
Real-Time



Limited resources

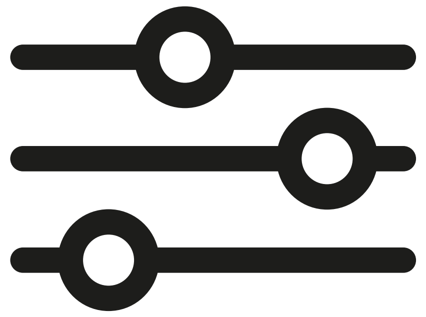


Reliability



Learning Convex Optimization Control Policies

Easy tuning

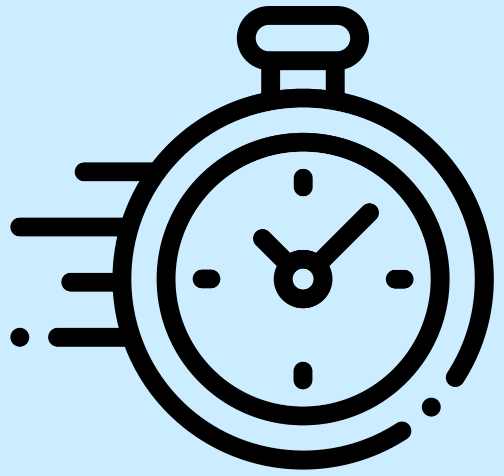


Today's talk

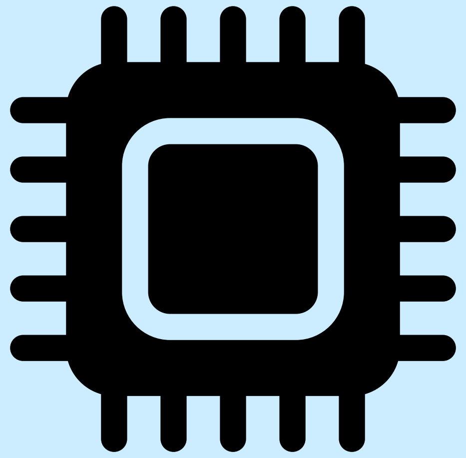
Data-Driven Embedded Optimization for Control

OSQP Solver

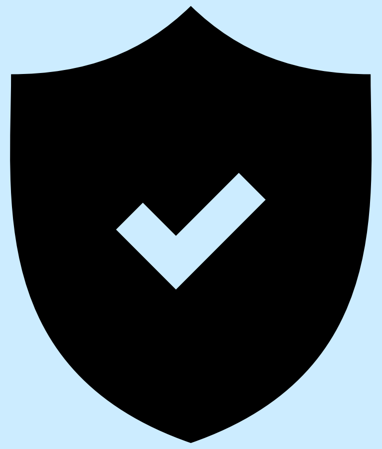
Real-Time



Limited resources

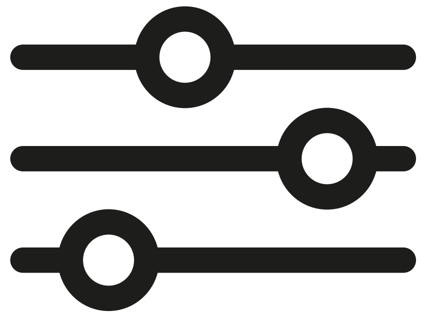


Reliability



Learning Convex Optimization Control Policies

Easy tuning



Still quadratic programming?

AN ALGORITHM FOR QUADRATIC PROGRAMMING

Marguerite Frank and Philip Wolfe[1]
Princeton University

A finite iteration method for calculating the solution of quadratic programming problems is described. Extensions to more general non-linear problems are suggested.

1. INTRODUCTION

The problem of maximizing a concave quadratic function whose variables are subject to linear inequality constraints has been the subject of several recent studies, from both the computational side and the theoretical (see Bibliography). Our aim here has been to develop a method for solving this non-linear programming problem which should be particularly well adapted to high-speed machine computation.

March 1956!


145
140
135
130
125
123.74
Tues.
142.78
Chg.
1.64
1.98
Day

First-order methods

Wide popularity

Pros

Warm-starting

Large-scale problems

Embeddable

First-order methods

Wide popularity

Pros	Cons
Warm-starting	Low quality solutions
Large-scale problems	Can't detect infeasibility
Embeddable	Problem data dependent

First-order methods

Wide popularity

Pros	Cons		OSQP
Warm-starting	Low quality solutions		High-quality solutions
Large-scale problems	Can’t detect infeasibility	→	Detects infeasibility
Embeddable	Problem data dependent		Robust

The problem

$$\begin{array}{ll}\text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & Ax \in \mathcal{C}\end{array}$$

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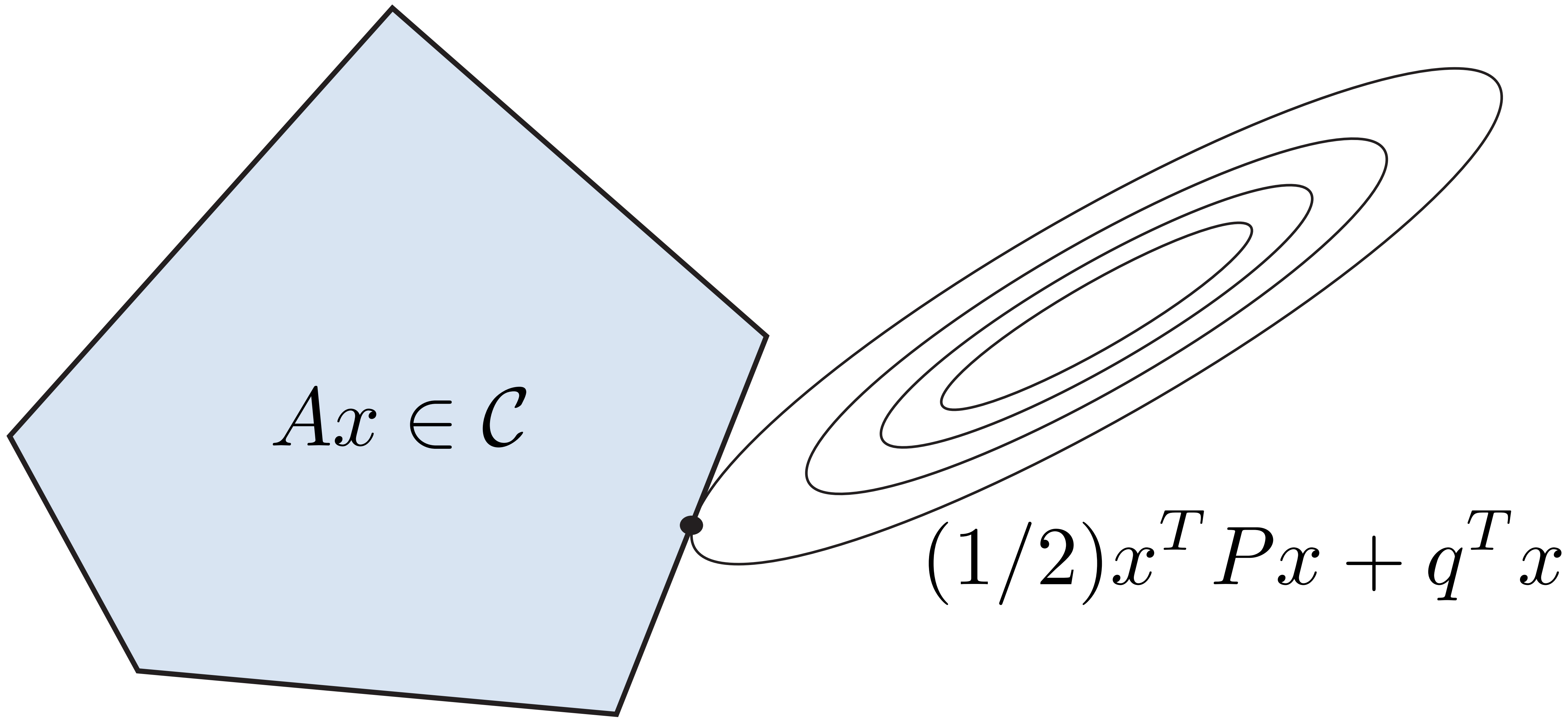
\end{array}
$$

Quadratic program: $\mathcal{C} = [l, u]$



ADMM

Alternating Direction Method of Multipliers

minimize $\quad f(x) + g(x)$ $\longrightarrow$

Splitting

minimize $\quad f(\tilde{x}) + g(x)$

subject to $\quad \tilde{x} = x$

ADMM

Alternating Direction Method of Multipliers

minimize $\quad f(x) + g(x)$ $\longrightarrow$

Splitting

$$\begin{aligned} &\text{minimize} && f(\tilde{x}) + g(x) \\ &\text{subject to} && \tilde{x} = x \end{aligned}$$

Iterations

$$\tilde{x}^{k+1} \leftarrow \underset{\tilde{x}}{\text{argmin}} \left(f(\tilde{x}) + \rho/2 \left\| \tilde{x} - (x^k - y^k/\rho) \right\|^2 \right)$$

$$x^{k+1} \leftarrow \underset{x}{\text{argmin}} \left(g(x) + \rho/2 \left\| x - (\tilde{x}^{k+1} + y^k/\rho) \right\|^2 \right)$$

$$y^{k+1} \leftarrow y^k + \rho \left(\tilde{x}^{k+1} - x^{k+1} \right)$$

How do we split the QP?

$$
\begin{array}{ll}
\text{minimize} & (1/2)x^T P x + q^T x \\
\text{subject to} & Ax = z \\
& z \in \mathcal{C}
\end{array}
$$

How do we split the QP?

$$
\begin{array}{ll}
\text{minimize} & \boxed{\begin{array}{l}(1/2)x^T P x + q^T x \\ Ax = z\end{array}} \\
\text{subject to} & \\
 & z \in \mathcal{C}
\end{array}
\qquad f
$$

How do we split the QP?

$$\begin{array}{lll} \text{minimize} & (1/2)x^T P x + q^T x & \multirow{2}{*}{f} \\ \text{subject to} & Ax = z & \\ & z \in \mathcal{C} & g \end{array}$$

How do we split the QP?

$$\begin{array}{lll} \text{minimize} & (1/2)x^T P x + q^T x & \\ \text{subject to} & Ax = z & f \\ & z \in \mathcal{C} & g \end{array}$$

Splitting formulation

$$\begin{array}{ll} \text{minimize} & \underbrace{(1/2)\tilde{x}^T P \tilde{x} + q^T \tilde{x} + \mathcal{I}_{Ax=z}(\tilde{x}, \tilde{z})}_{f} + \underbrace{\mathcal{I}_{\mathcal{C}}(z)}_{g} \\ \text{subject to} & \tilde{x} = x \\ & \tilde{z} = z \end{array}$$

How do we split the QP?

$$
\begin{array}{lll}
\text{minimize} & (1/2)x^T P x + q^T x & \\
\text{subject to} & Ax = z & f \\
 & z \in \mathcal{C} & g
\end{array}
$$

Splitting formulation

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\begin{array}{ll}
\text{minimize} & \overbrace{(1/2)\tilde{x}^T P \tilde{x} + q^T \tilde{x} + \mathcal{I}_{Ax=z}(\tilde{x}, \tilde{z})}^{f} + \overbrace{\mathcal{I}_{\mathcal{C}}(z)}^{g} \\
\text{subject to} & \tilde{x} = x \\
 & \tilde{z} = z
\end{array}
$$

Step sizes

$\sigma > 0$

$\rho > 0$

ADMM iterations

$$(x^{k+1}, \tilde{z}^{k+1}) \leftarrow \underset{(x,z):Ax=z}{\operatorname{argmin}} (1/2)x^T P x + q^T x + \sigma/2 \left\| x - x^k \right\|^2 + \rho/2 \left\| z - z^k + y^k/\rho \right\|^2$$

$$z^{k+1} \leftarrow \Pi \left(\tilde{z}^{k+1} + y^k/\rho \right)$$

$$y^{k+1} \leftarrow y^k + \rho \left(\tilde{z}^{k+1} - z^{k+1} \right)$$

ADMM iterations

Inner QP

$$(x^{k+1}, \tilde{z}^{k+1}) \leftarrow \underset{(x,z):Ax=z}{\operatorname{argmin}} \ (1/2)x^T P x + q^T x + \sigma/2 \left\| x - x^k \right\|^2 + \rho/2 \left\| z - z^k + y^k/\rho \right\|^2$$

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ADMM iterations

Inner QP

$$(x^{k+1}, \tilde{z}^{k+1}) \leftarrow \underset{(x,z):Ax=z}{\operatorname{argmin}} \; (1/2)x^T P x + q^T x + \sigma/2 \left\| x - x^k \right\|^2 + \rho/2 \left\| z - z^k + y^k/\rho \right\|^2$$

$$z^{k+1} \leftarrow \Pi\left(\tilde{z}^{k+1} + y^k/\rho\right)$$

Projection onto $\mathcal{C}$

$$y^{k+1} \leftarrow y^k + \rho\left(\tilde{z}^{k+1} - z^{k+1}\right)$$

Solving the inner QP

Equality-constrained

$$
\begin{array}{ll}
\text{minimize} & (1/2)x^T P x + q^T x + \sigma/2 \left\| x - x^k \right\|^2 + \rho/2 \left\| z - z^k + y^k/\rho \right\|^2 \\
\text{subject to} & Ax = z
\end{array}
$$

Solving the inner QP

Equality-constrained

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x + \sigma/2 \left\| x - x^k \right\|^2 + \rho/2 \left\| z - z^k + y^k/\rho \right\|^2 \\ \text{subject to} & Ax = z \end{array}$$

Reduced KKT system

$$\begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

Solving the inner QP

Equality-constrained

$$
\begin{array}{ll}
\text{minimize} & (1/2)x^T P x + q^T x + \sigma/2 \left\| x - x^k \right\|^2 + \rho/2 \left\| z - z^k + y^k/\rho \right\|^2 \\
\text{subject to} & Ax = z
\end{array}
$$

Reduced KKT system

Always solvable!

$$
\begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}
$$

Solving the linear system

Direct method (small to medium scale)

$$\begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

Solving the linear system

Direct method (small to medium scale)

Quasi-definite matrix

$$\begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

Solving the linear system

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Well-defined LDL^T factorization

Factorization caching

Solving the linear system

Direct method (small to medium scale)

Quasi-definite matrix

$$\begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

Well-defined LDL^T factorization

Factorization caching → **QDLDL Free quasi-definite linear system solver**

[https://github.com/oxfordcontrol/qdldl]

Solving the linear system

Indirect method (large scale)

$$\left(P + \sigma I + \rho A^T A\right) x = \sigma x^k - q + A^T(\rho z^k - y^k)$$

Solving the linear system

Indirect method (large scale)

Positive-definite matrix

$$\left(P + \sigma I + \rho A^T A\right) x = \sigma x^k - q + A^T(\rho z^k - y^k)$$

Solving the linear system

Indirect method (large scale)

Positive-definite matrix

$$\left(P + \sigma I + \rho A^T A\right) x = \sigma x^k - q + A^T(\rho z^k - y^k)$$

Conjugate gradient

Solve very large systems

Solving the linear system

Indirect method (large scale)

Positive-definite matrix

$$\boxed{\left(P + \sigma I + \rho A^T A\right)} x = \sigma x^k - q + A^T(\rho z^k - y^k)$$

Conjugate gradient

Solve very large systems

→

GPU implementation

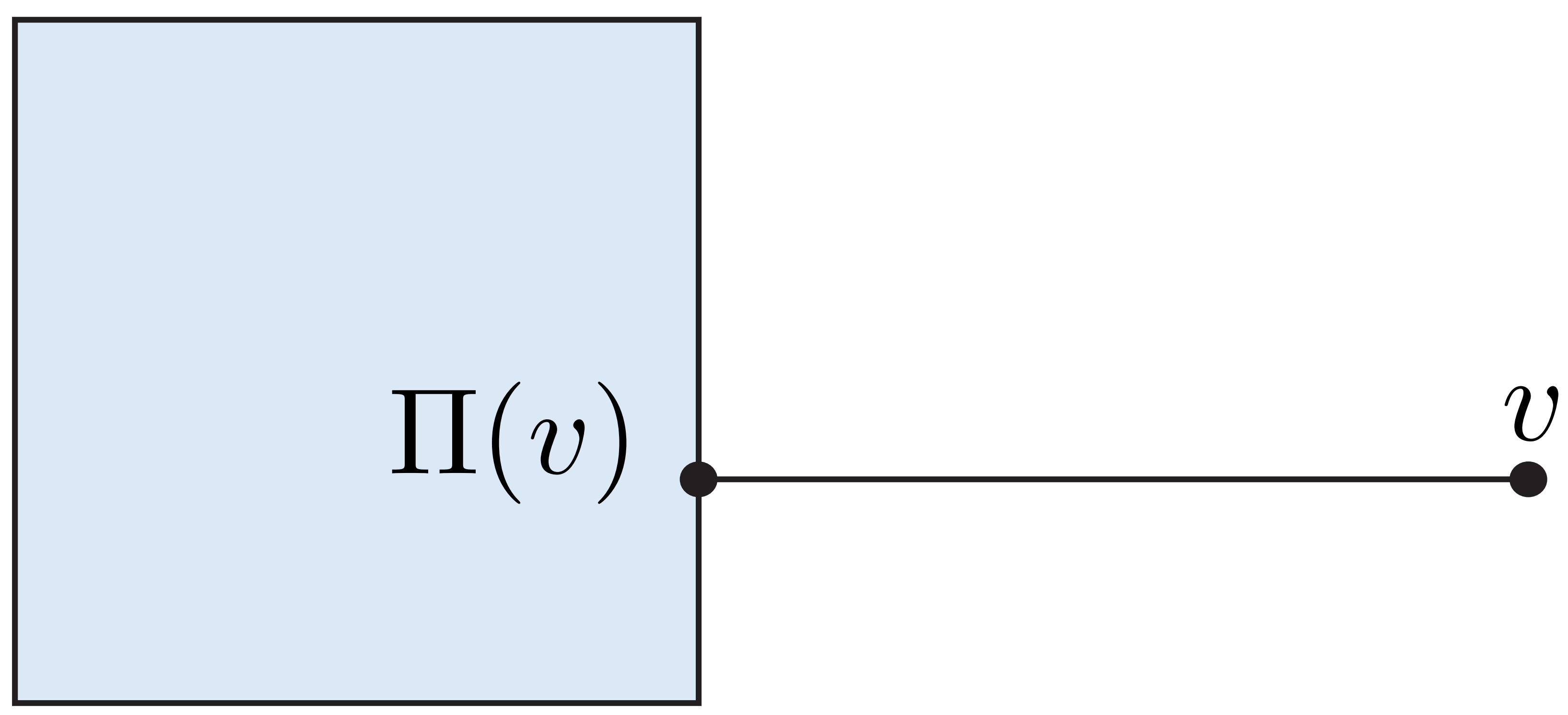
[https://github.com/oxfordcontrol/cuosqp]

Computing the projection

Quadratic program: $\mathcal{C} = [l, u]$

Box projection

$$\Pi(v) = \max(\min(v, u), l)$$



Complete algorithm

Problem

$$
\begin{array}{ll}
\text{minimize} & (1/2)x^T P x + q^T x \\
\text{subject to} & l \leq Ax \leq u
\end{array}
$$

Complete algorithm

Problem

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & l \leq Ax \leq u \end{array}$$

Algorithm

$$(x^{k+1}, \nu^{k+1}) \leftarrow \text{solve} \begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x^{k+1} \\ \nu^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

$$\tilde{z}^{k+1} \leftarrow z^k + (\nu^{k+1} - y^k)/\rho$$

$$z^{k+1} \leftarrow \Pi\left(\tilde{z}^{k+1} + y^k/\rho\right)$$

$$y^{k+1} \leftarrow y^k + \rho\left(\tilde{z}^{k+1} - z^{k+1}\right)$$

Complete algorithm

Problem

minimize $\quad (1/2)x^T P x + q^T x$

subject to $\quad l \leq Ax \leq u$

Algorithm

Linear system solve

$$(x^{k+1}, \nu^{k+1}) \leftarrow \text{solve} \begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x^{k+1} \\ \nu^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

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Complete algorithm

Problem

$$\begin{array}{ll}\text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & l \leq Ax \leq u\end{array}$$

Algorithm

Linear system solve

$$(x^{k+1}, \nu^{k+1}) \leftarrow \text{solve} \begin{bmatrix} P + \sigma I & A^T \\ A & -\frac{1}{\rho} I \end{bmatrix} \begin{bmatrix} x^{k+1} \\ \nu^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \frac{1}{\rho} y^k \end{bmatrix}$$

Easy operations

$$\tilde{z}^{k+1} \leftarrow z^k + (\nu^{k+1} - y^k)/\rho$$

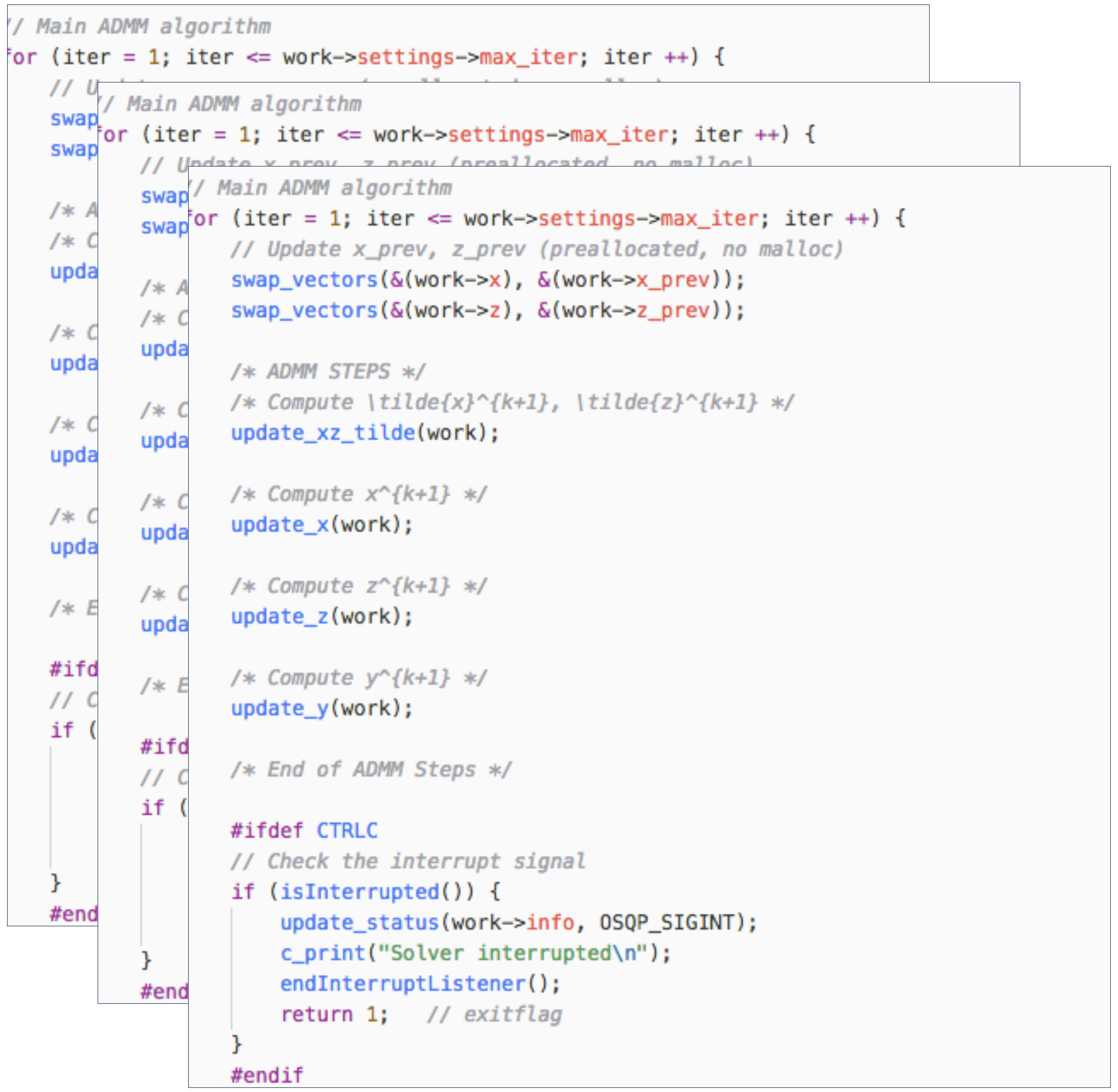
$$z^{k+1} \leftarrow \Pi\left(\tilde{z}^{k+1} + y^k/\rho\right)$$

$$y^{k+1} \leftarrow y^k + \rho\left(\tilde{z}^{k+1} - z^{k+1}\right)$$

Code generation

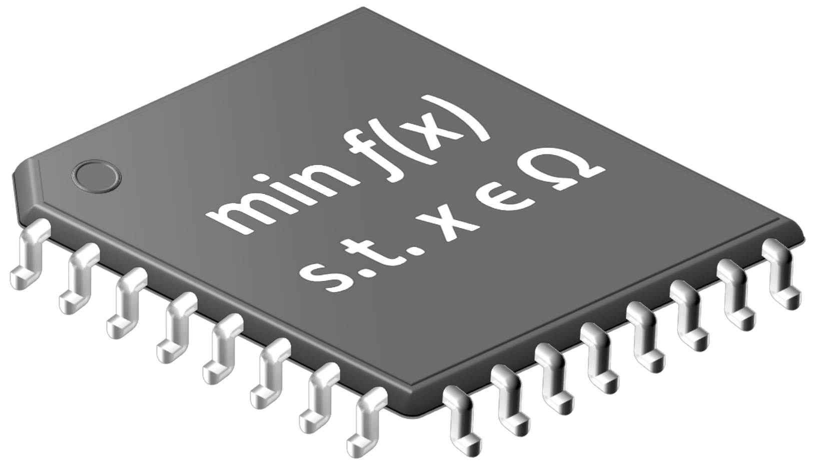
```python
# Create OSQP object
m = osqp.OSQP()

# Initialize solver
m.setup(P, q, A, l, u,
        settings)

# Generate C code
m.codegen('folder_name')
```

Optimized C code



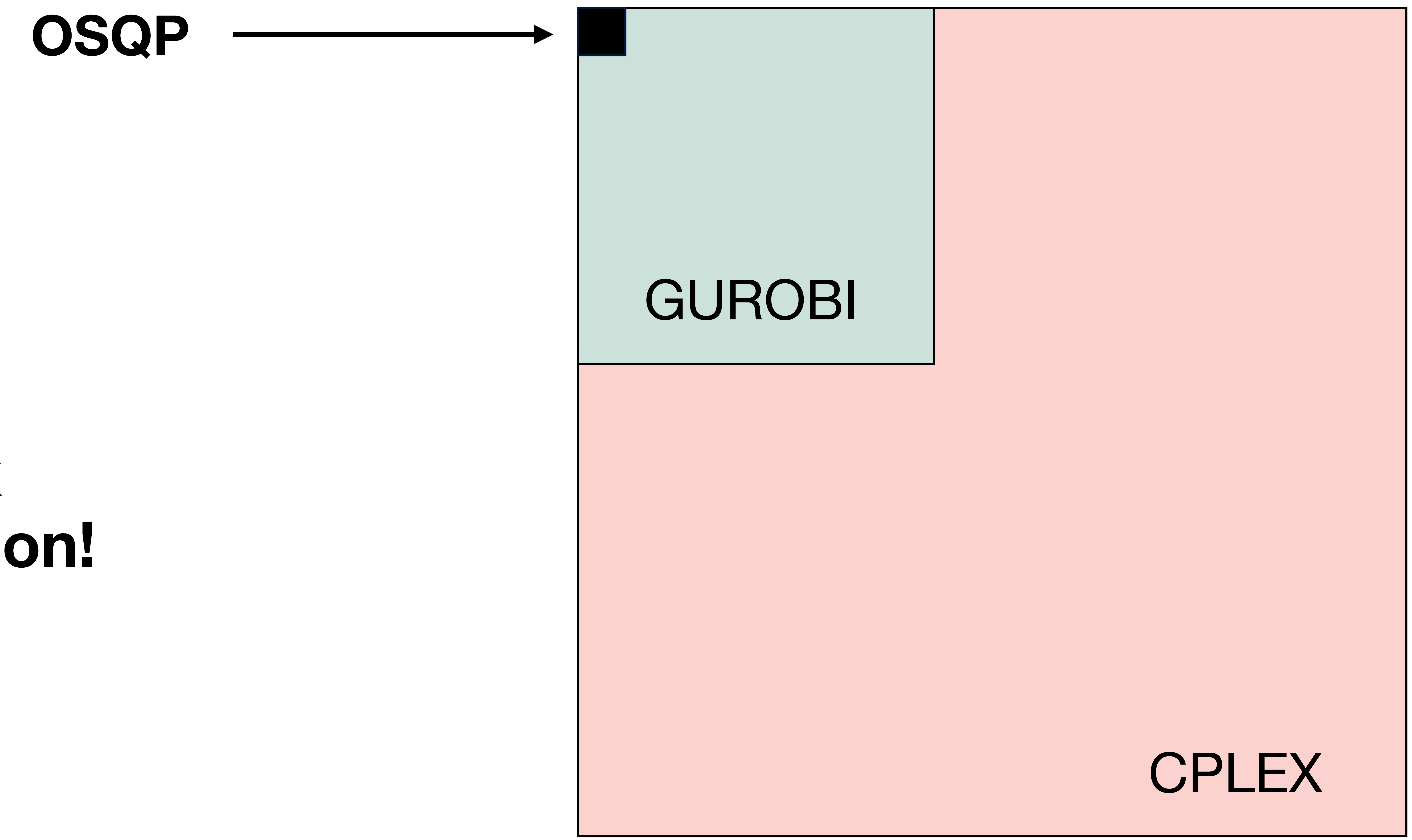
```c
// Main ADMM algorithm
for (iter = 1; iter <= work->settings->max_iter; iter ++) {
    // Update x_prev, z_prev (preallocated, no malloc)
    swap_vectors(&(work->x), &(work->x_prev));
    swap_vectors(&(work->z), &(work->z_prev));

    /* ADMM STEPS */
    /* Compute \tilde{x}^{k+1}, \tilde{z}^{k+1} */
    update_xz_tilde(work);

    /* Compute x^{k+1} */
    update_x(work);

    /* Compute z^{k+1} */
    update_z(work);

    /* Compute y^{k+1} */
    update_y(work);

    /* End of ADMM Steps */

    #ifdef CTRLC
    // Check the interrupt signal
    if (isInterrupted()) {
        update_status(work->info, OSQP_SIGINT);
        c_print("Solver interrupted\n");
        endInterruptListener();
        return 1;   // exitflag
    }
    #endif
```

Embedded Hardware



It can be compiled into division-free

Compiled code size ~80kb (low footprint)



300x Reduction!

How do we ensure fast convergence?

$$
\begin{array}{ll}
\text{minimize} & (1/2)x^T P x + q^T x \\
\text{subject to} & Ax = z \\
 & l \le z \le u
\end{array}
$$

Primal residual

$$r^k_{\text{prim}} = Ax^k - z^k$$

Dual residual

$$r^k_{\text{dual}} = Px^k + q + A^T y^k$$

How do we ensure fast convergence?

minimize $\quad (1/2)x^T P x + q^T x$

subject to $\quad Ax = z$

$\quad\quad l \leq z \leq u$

Primal residual

$$r_{\text{prim}}^k = Ax^k - z^k$$

Dual residual

$$r_{\text{dual}}^k = Px^k + q + A^T y^k$$

Linear system in indirect method

$$\left(P + \rho A^T A\right) x^{k+1} = -q + A^T(\rho z^k - y^k)$$

How do we ensure fast convergence?

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & Ax = z \\ & l \le z \le u \end{array}$$

Primal residual

$$r_{\text{prim}}^k = Ax^k - z^k$$

Dual residual

$$r_{\text{dual}}^k = Px^k + q + A^T y^k$$

Linear system in indirect method

$$\left(P + \rho A^T A\right) x^{k+1} = -q + A^T(\rho z^k - y^k)$$

$$\rho = \infty$$

$$A^T A x^{k+1} = A^T z^k$$

Small primal residual

How do we ensure fast convergence?

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & Ax = z \\ & l \leq z \leq u \end{array}$$

Primal residual

$$r^k_{\text{prim}} = Ax^k - z^k$$

Dual residual

$$r^k_{\text{dual}} = Px^k + q + A^T y^k$$

Linear system in indirect method

$$\left(P + \rho A^T A\right) x^{k+1} = -q + A^T(\rho z^k - y^k)$$

$\rho = \infty$

$$A^T A x^{k+1} = A^T z^k$$

Small primal residual

$\rho = 0$

$$P x^{k+1} = -q - A^T y^k$$

Small dual residual

How do we ensure fast convergence?

$$
\begin{array}{ll}
\text{minimize} & (1/2)x^T P x + q^T x \\
\text{subject to} & Ax = z \\
& l \leq z \leq u
\end{array}
$$

Primal residual

$$r_{\mathrm{prim}}^k = Ax^k - z^k$$

Dual residual

$$r_{\mathrm{dual}}^k = Px^k + q + A^T y^k$$

Linear system in indirect method

$$\left(P + \rho A^T A\right) x^{k+1} = -q + A^T(\rho z^k - y^k)$$

$\rho = \infty$

$\rho = 0$

$$A^T A x^{k+1} = A^T z^k$$

Small primal residual

$$P x^{k+1} = -q - A^T y^k$$

Small dual residual

What's the optimal ρ?

Extreme cases

Equality constrained QP

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & Ax = b \end{array}$$

$\longrightarrow$

Solve in one ADMM step

$$\rho \approx \infty$$

$$\begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} -q \\ b \end{bmatrix}$$

Extreme cases

Solve in one ADMM step

Equality constrained QP

$$\rho \approx \infty$$

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & Ax = b \end{array} \longrightarrow \begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} -q \\ b \end{bmatrix}$$

Unconstrained QP

Solve in one ADMM step

$$\rho \approx 0$$

$$\text{minimize} \quad (1/2)x^T P x + q^T x \longrightarrow Px = -q$$

Extreme cases

Equality constrained QP

minimize $\quad (1/2)x^T P x + q^T x$
subject to $\quad Ax = b$

$\longrightarrow$

Solve in one ADMM step

$$\rho \approx \infty$$

$$\begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \nu \end{bmatrix} = \begin{bmatrix} -q \\ b \end{bmatrix}$$

Unconstrained QP

minimize $\quad (1/2)x^T P x + q^T x$

$\longrightarrow$

Solve in one ADMM step

$$\rho \approx 0$$

$$Px = -q$$

We need different step sizes

Constraint-wise step size

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & l \leq Ax \leq u \end{array}$$

Tight constraints

$l_i = (Ax^\star)_i$ or $(Ax^\star)_i = u_i$

Constraint-wise step size

minimize $\quad (1/2)x^T Px + q^T x$

subject to $\quad l \le Ax \le u$

Tight constraints

$l_i = (Ax^\star)_i$ or $(Ax^\star)_i = u_i$

$\rho = (\rho_1, \ldots, \rho_m)$ **can be a vector**

Constraint-wise step size

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$\rho = (\rho_1, \ldots, \rho_m)$ **can be a vector**

Never tight

$l_i = -\infty$ and $u_i = \infty$

$$\downarrow$$

$\rho_i = 0$

Constraint-wise step size

minimize $\quad (1/2)x^T Px + q^T x$

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Tight constraints

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Never tight

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$$\downarrow$$

$\rho_i = 0$

Always tight

$l_i = u_i \neq \infty$

$$\downarrow$$

$\rho_i = \infty$

Constraint-wise step size

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Tight constraints

$l_i = (Ax^\star)_i$ or $(Ax^\star)_i = u_i$

$\rho = (\rho_1, \ldots, \rho_m)$ **can be a vector**

Never tight

$l_i = -\infty$ and $u_i = \infty$

$$\downarrow$$

$$\rho_i = 0$$

l_i or u_i finite

$$\downarrow$$

Balance residuals

$$\rho_i^{k+1} \leftarrow \rho_i^k \sqrt{\|r_{\text{prim}}\| / \|r_{\text{dual}}\|}$$

Always tight

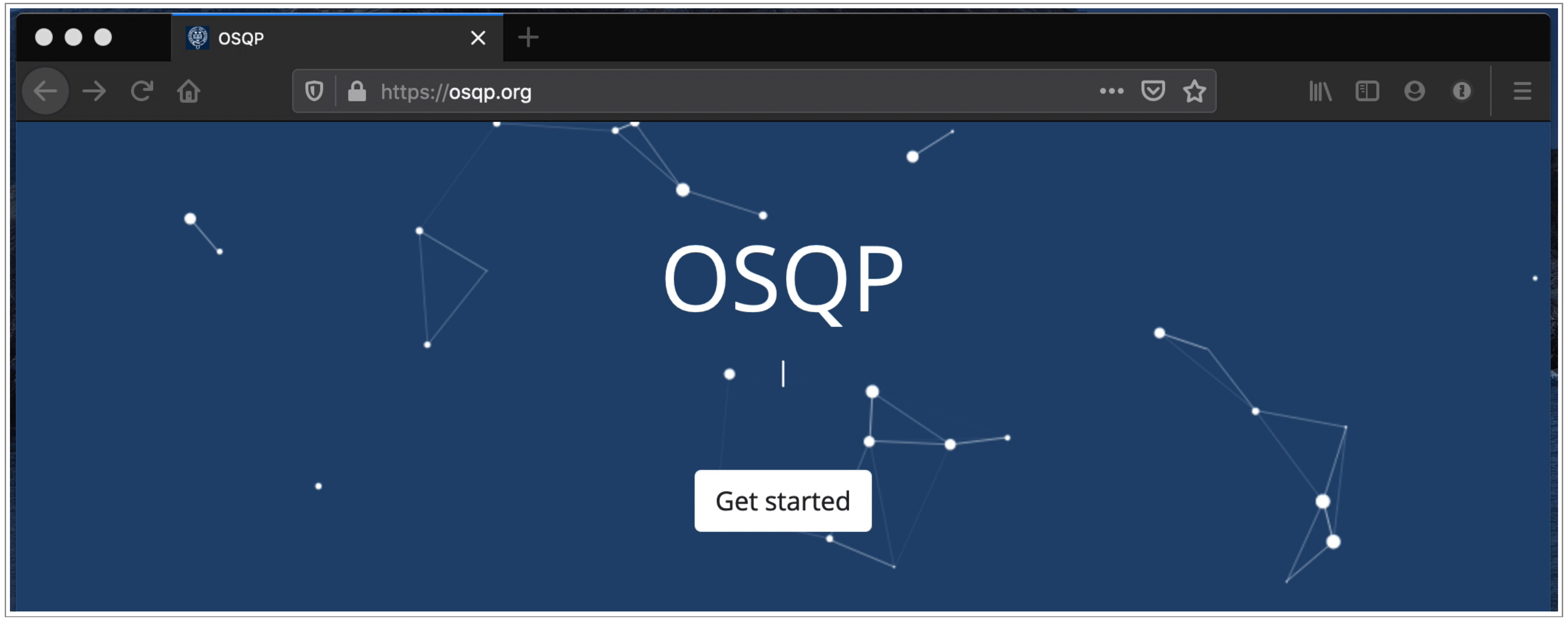
$l_i = u_i \neq \infty$

$$\downarrow$$

$$\rho_i = \infty$$

OSQP

Operator Splitting solver for Quadratic Programs



Embeddable (can be division free!)

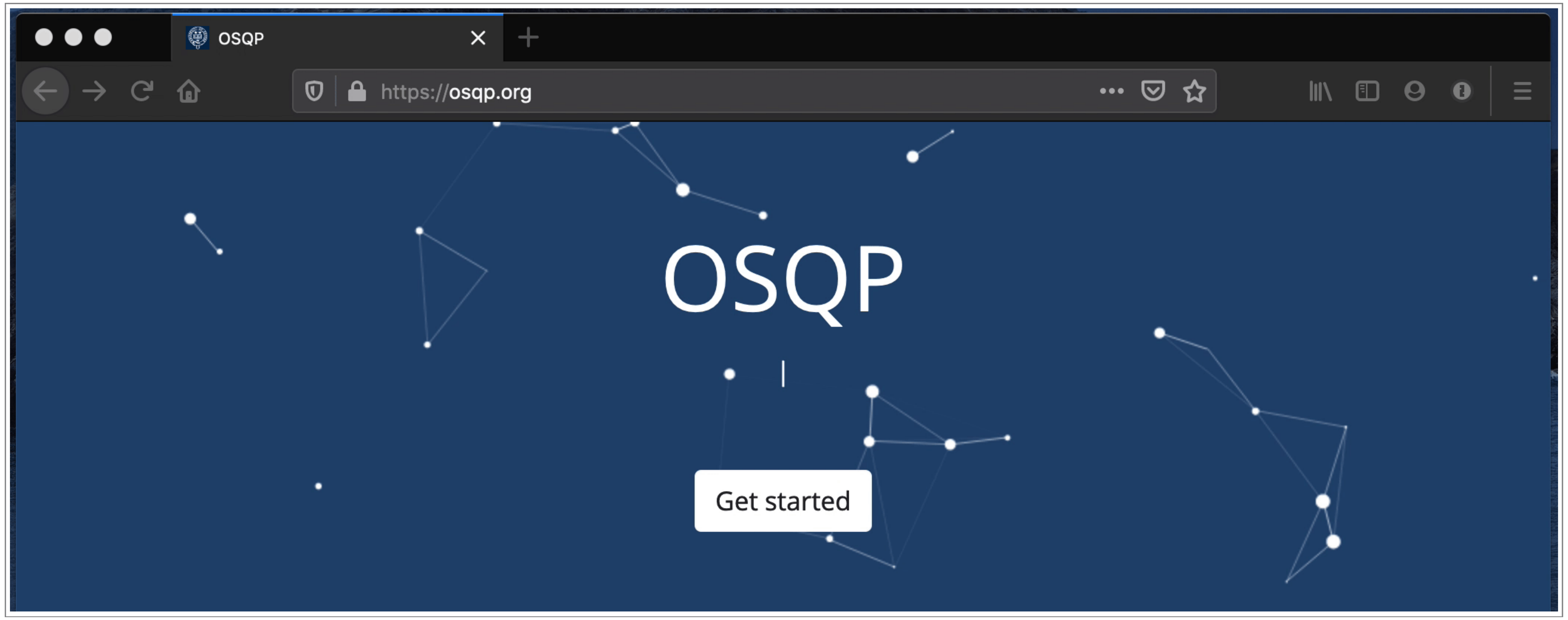
Supports warm-starting

Detects infeasibility

Solves large-scale problems

OSQP

Operator Splitting solver for Quadratic Programs



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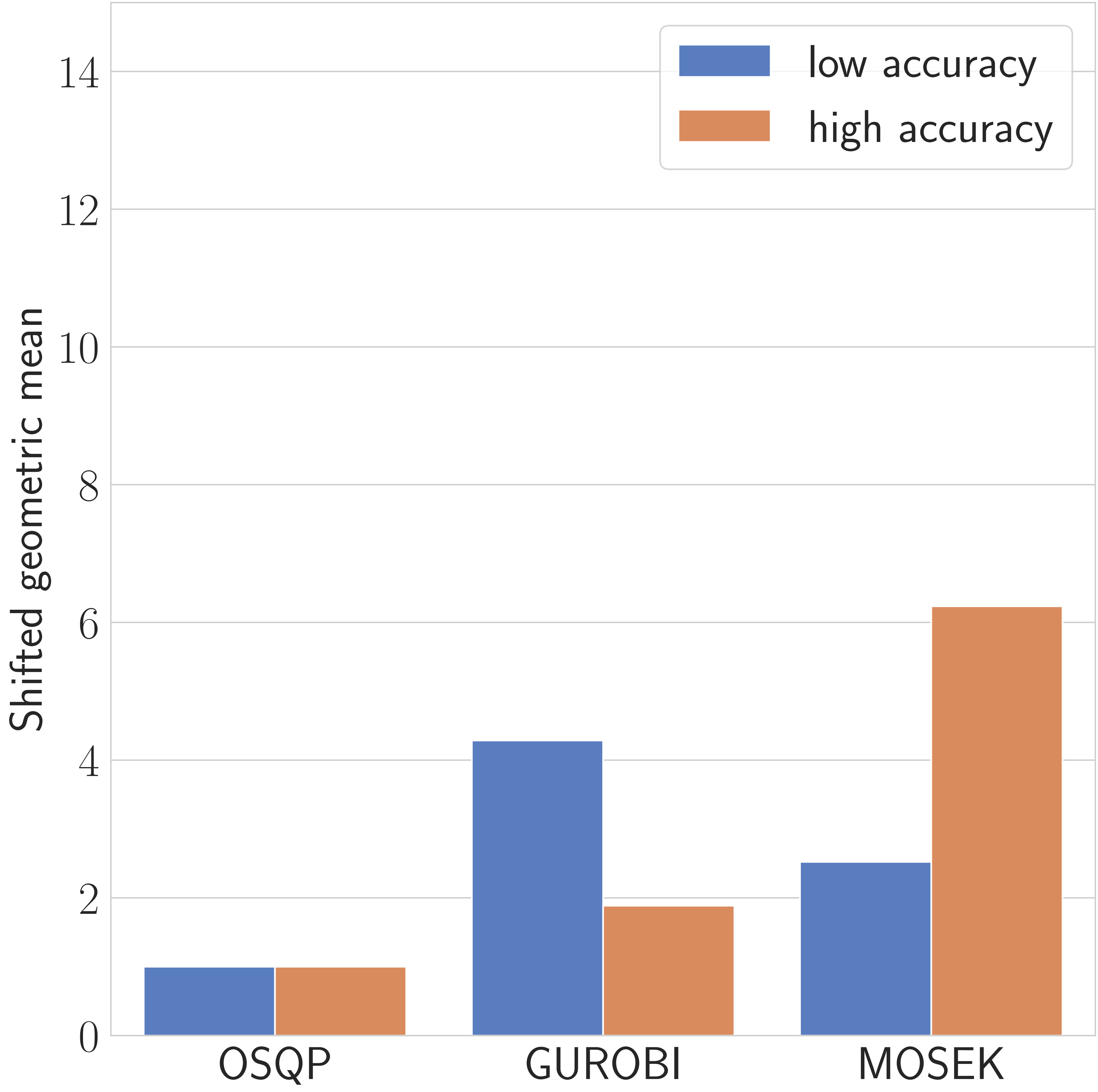
Users

More than 7 million downloads!

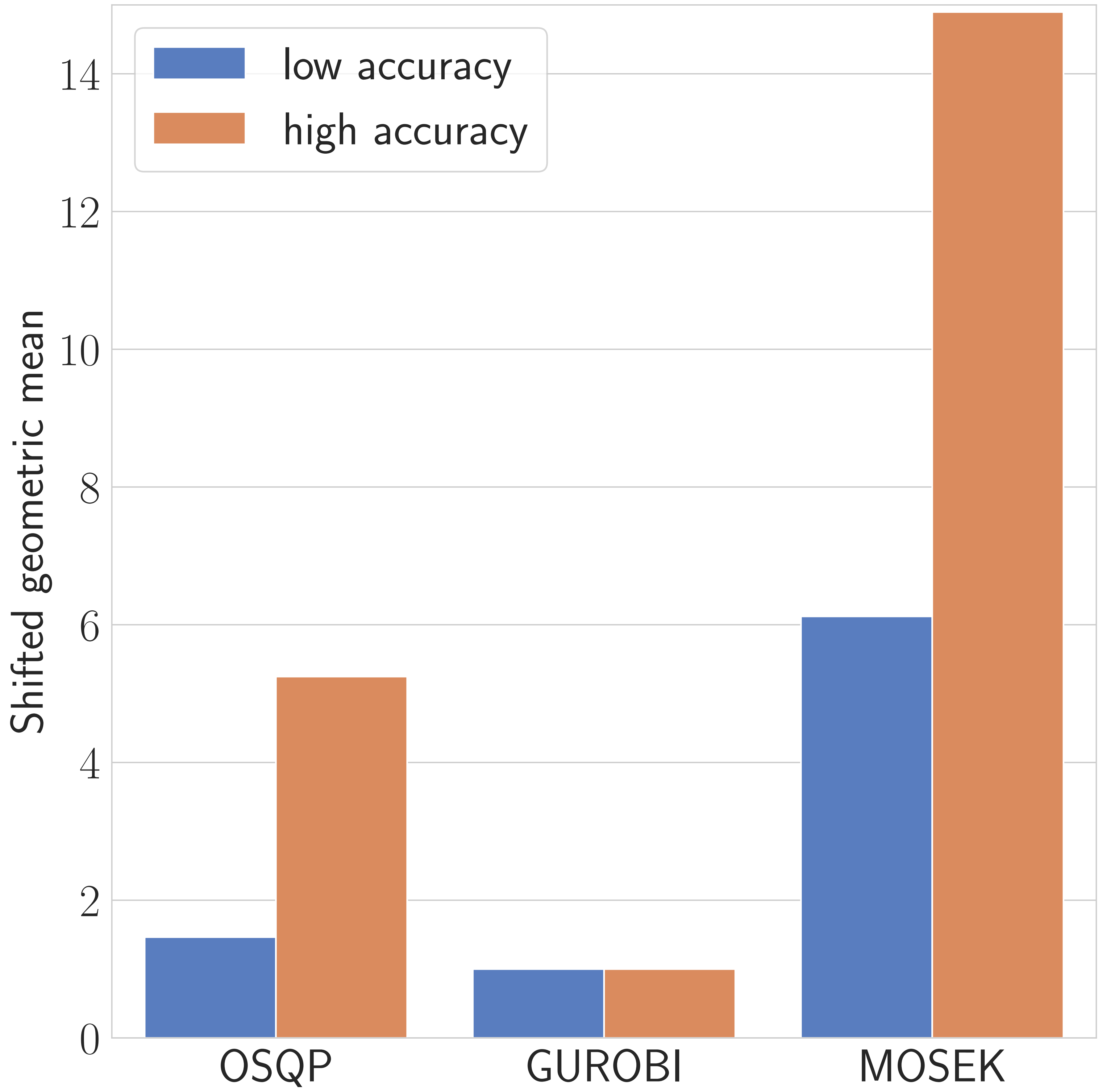
[pepy.tech/project/osqp]

Performance benchmarks

OSQP Benchmarks
(control, portfolio, lasso, SVM, etc.)



Maroz-Meszaros



[github.com/oxfordcontrol/osqp_benchmarks]

Constraint-wise step size

minimize $\quad (1/2)x^T Px + q^T x$

subject to $\quad l \leq Ax \leq u$

Tight constraints

$l_i = (Ax^\star)_i$ or $(Ax^\star)_i = u_i$

Vector step size

$\rho = (\rho_1, \ldots, \rho_m)$

Never tight

$l_i = -\infty$ and $u_i = \infty$

$\downarrow$

$\rho_i = 0$

Otherwise

$\downarrow$

Balance residuals

$\rho_i^{k+1} \leftarrow \rho_i^k \sqrt{\|r_{\text{prim}}\| / \|r_{\text{dual}}\|}$

Always tight

$l_i = u_i \neq \infty$

$\downarrow$

$\rho_i = \infty$

Constraint-wise step size

$$\begin{array}{ll} \text{minimize} & (1/2)x^T P x + q^T x \\ \text{subject to} & l \leq Ax \leq u \end{array}$$

Tight constraints

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Vector step size

$\rho = (\rho_1, \ldots, \rho_m)$

Never tight

$l_i = -\infty$ and $u_i = \infty$

$$\downarrow$$

$\rho_i = 0$

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$$\downarrow$$

Balance residuals

$$\rho_i^{k+1} \leftarrow \rho_i^k \sqrt{\|r_{\text{prim}}\| / \|r_{\text{dual}}\|}$$

Always tight

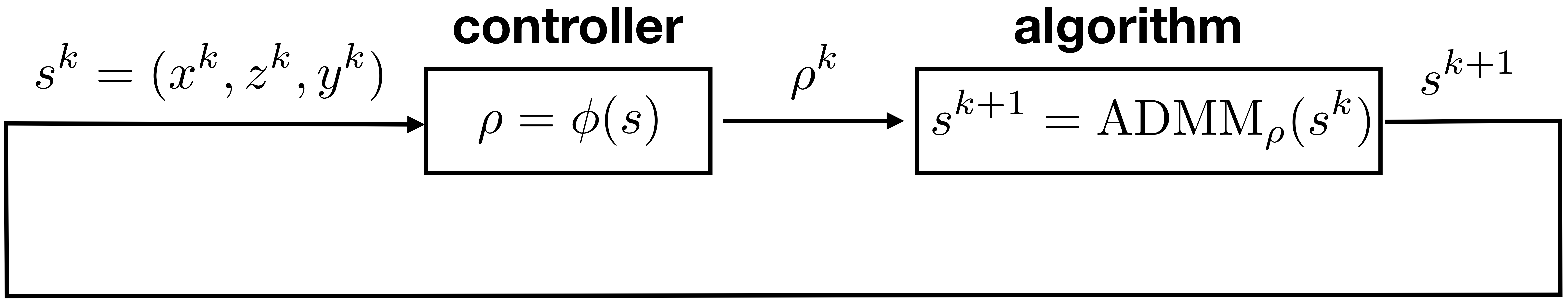
$l_i = u_i \neq \infty$

$$\downarrow$$

$\rho_i = \infty$

Can we learn a better update rule from data?

Step size choice as a control problem



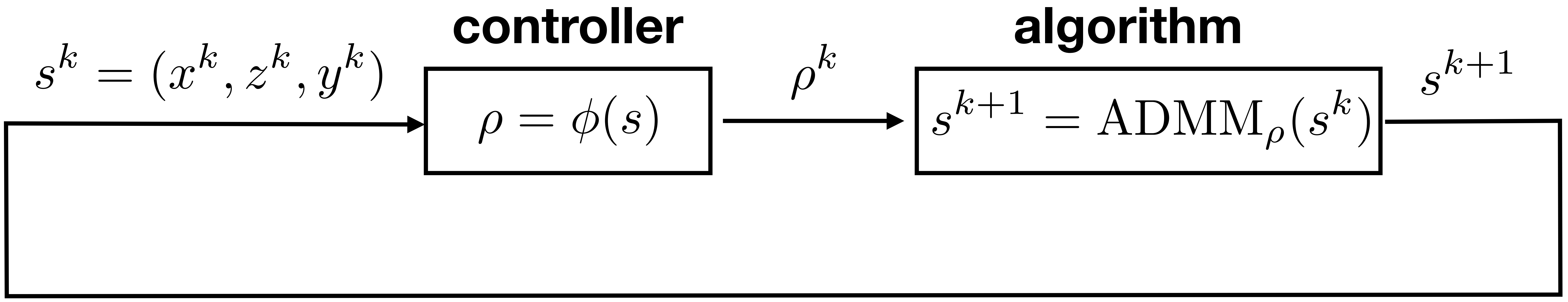
Stage cost

$$\ell(s) = \begin{cases} 1 & \text{if not converged} \\ 0 & \text{if converged} \end{cases}$$

Cumulative cost

$$J = \mathbf{E} \sum_{k=1}^{\infty} \gamma^k \ell(s^k)$$

Step size choice as a control problem



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Cumulative cost

$$J = \mathbf{E} \sum_{k=1}^{\infty} \gamma^k \ell(s^k)$$

Train with
Deep Policy Gradient methods (TD3)

Constraint-wise control policy

$$\phi(s) = \begin{bmatrix} \phi_c(s_1) \\ \phi_c(s_2) \\ \vdots \\ \phi_c(s_m) \end{bmatrix}$$

Per-constraint update rule

$$\rho_i = \phi_c(s_i)$$

Constraint-wise control policy

$$\phi(s) = \begin{bmatrix} \phi_c(s_1) \\ \phi_c(s_2) \\ \vdots \\ \phi_c(s_m) \end{bmatrix}$$

Per-constraint update rule

$$\rho_i = \phi_c(s_i)$$

Per-constraint state

$$s_i = \begin{bmatrix} \min(z_i - l_i, u_i - z_i) \\ (Ax)_i - z_i \\ y_i \end{bmatrix}$$

Constraint-wise control policy

$$\phi(s) = \begin{bmatrix} \phi_c(s_1) \\ \phi_c(s_2) \\ \vdots \\ \phi_c(s_m) \end{bmatrix}$$

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slacks

Constraint-wise control policy

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Per-constraint state

$$s_i = \begin{bmatrix} \min(z_i - l_i, u_i - z_i) \\ (Ax)_i - z_i \\ y_i \end{bmatrix} \begin{matrix} \textbf{slacks} \\ \textbf{infeasibility} \\ \end{matrix}$$

Constraint-wise control policy

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Constraint-wise control policy

$$\phi(s) = \begin{bmatrix} \phi_c(s_1) \\ \phi_c(s_2) \\ \vdots \\ \phi_c(s_m) \end{bmatrix}$$

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Per-constraint state

$$s_i = \begin{bmatrix} \min(z_i - l_i, u_i - z_i) \\ (Ax)_i - z_i \\ y_i \end{bmatrix}$$

slacks

infeasibility

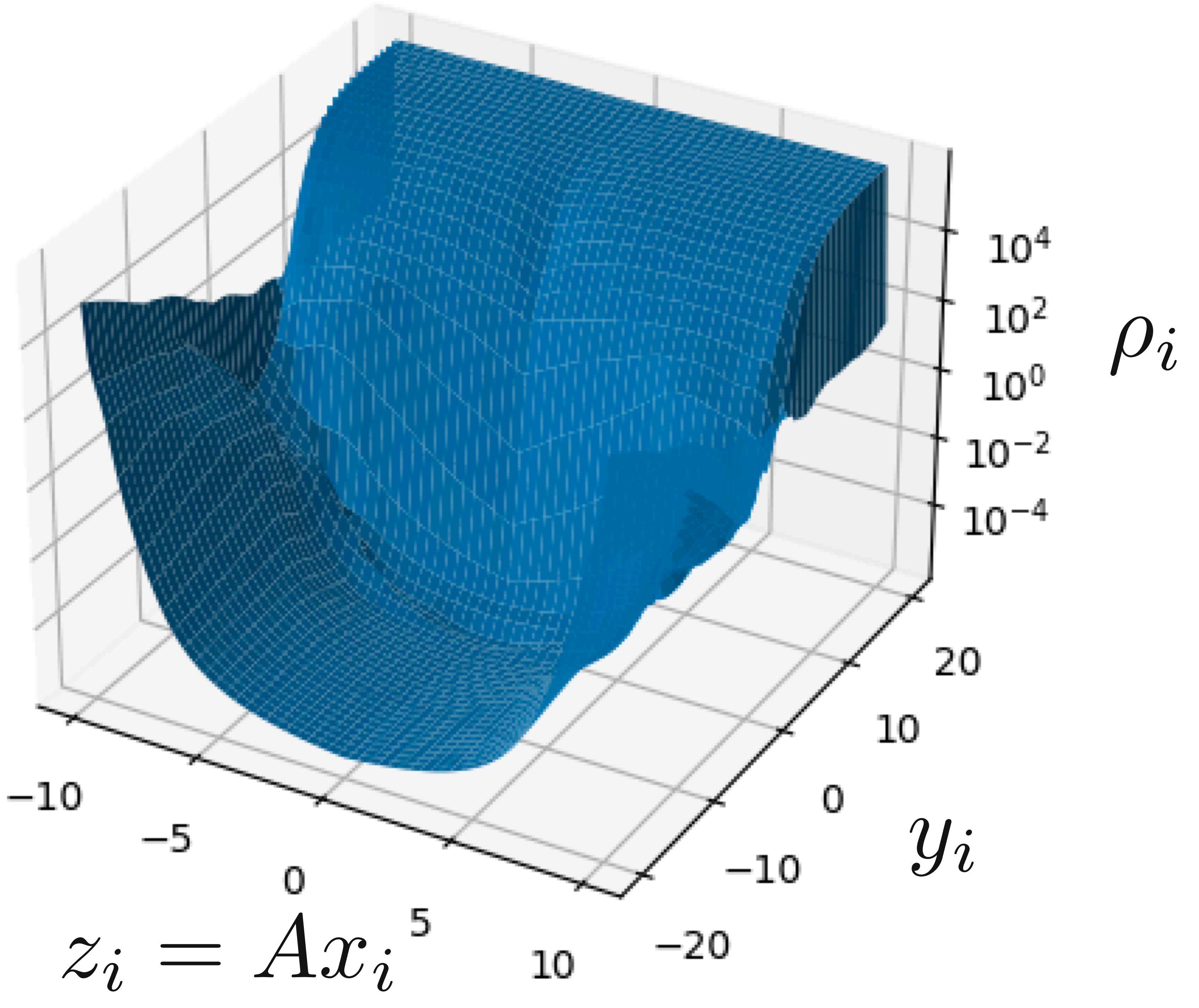
dual variable

Generalize to different dimensions

Low-dimensional state per constraint

Small NN policy $\phi_c(s_i)$

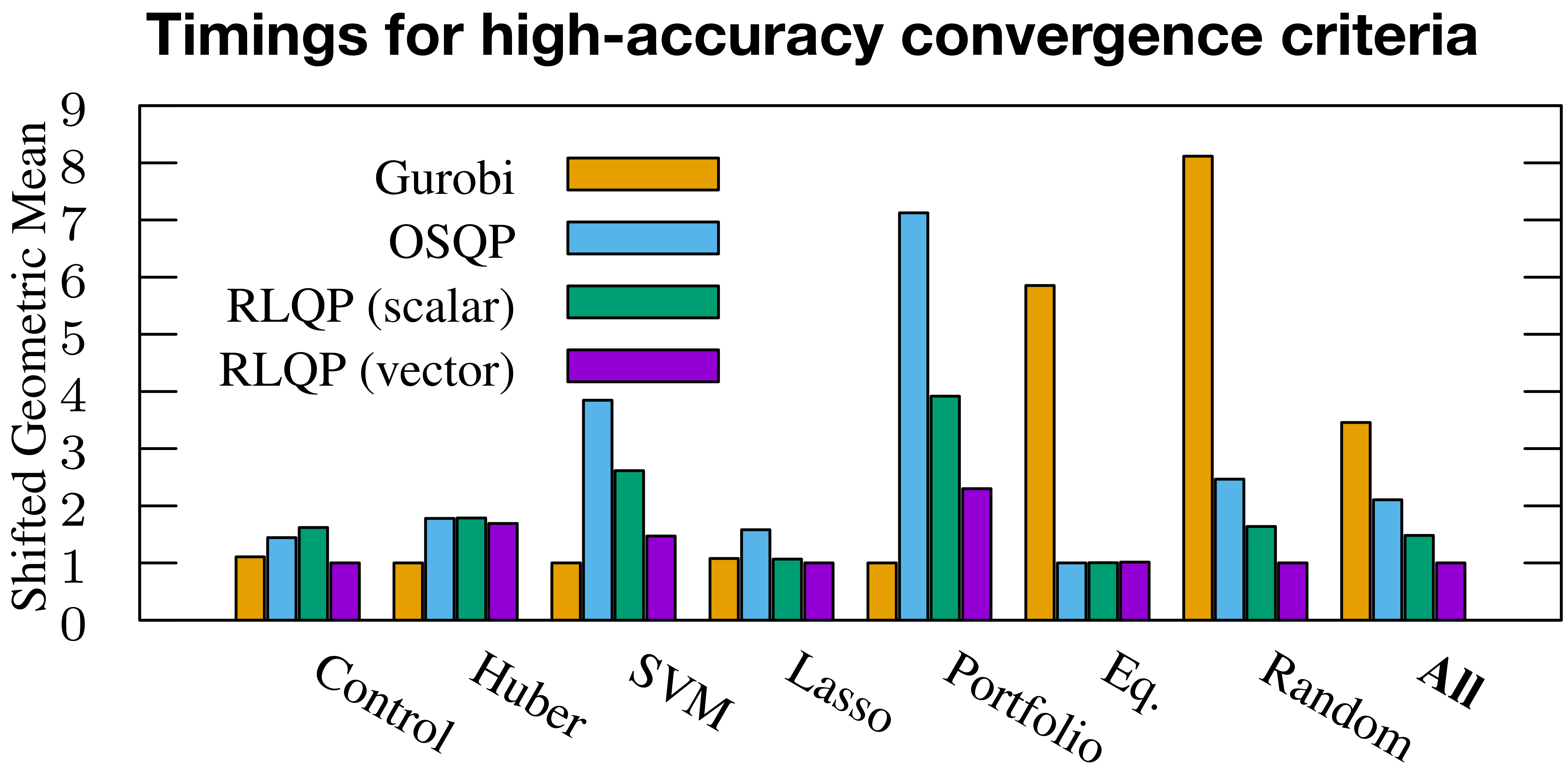
Visualize learned policy



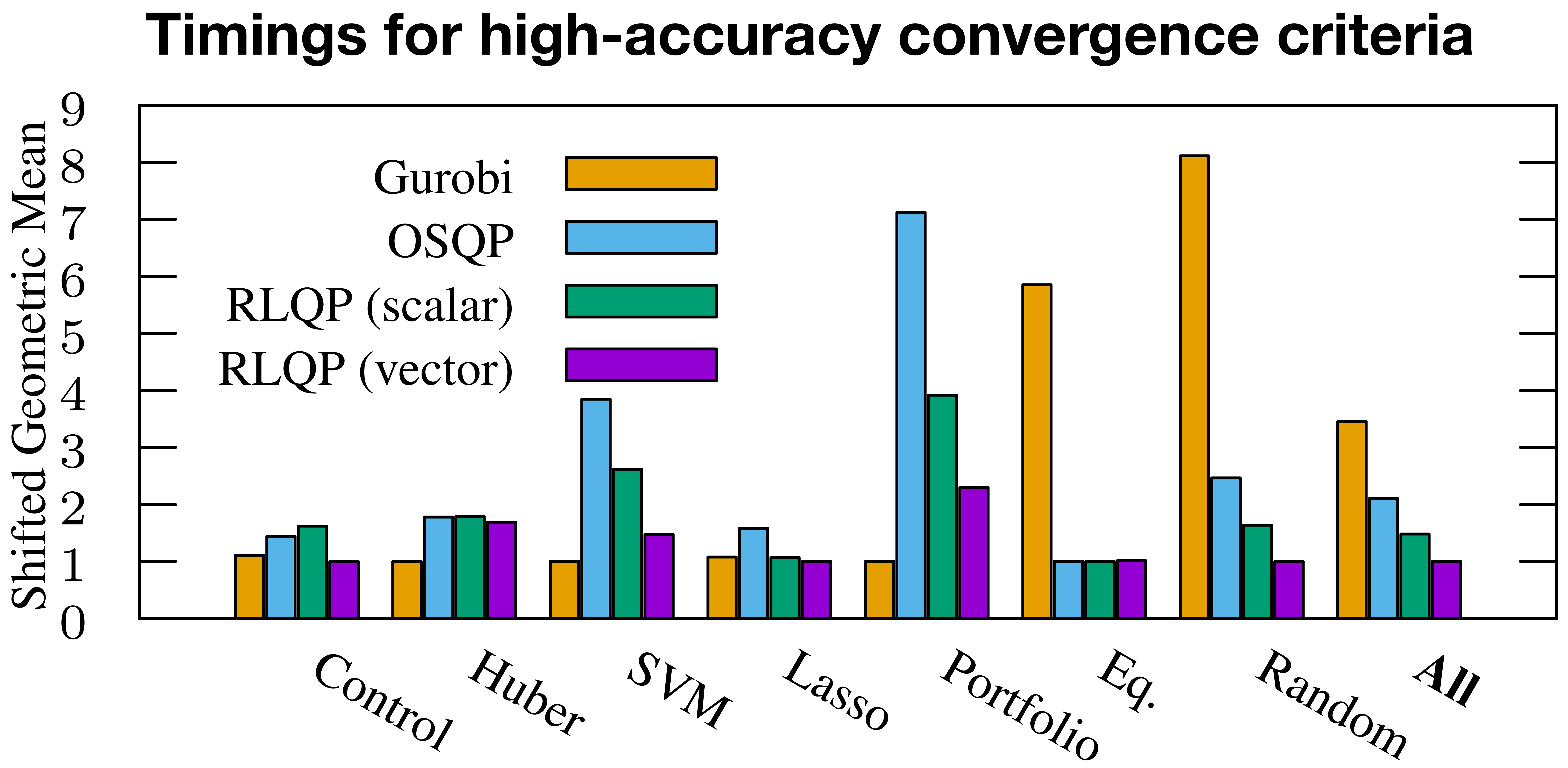
Interpretable policy

High step size ρ_i
when we reach bounds
$z_i \approx l_i$ and $z_i \approx u_i$.

Performance with step size learning



Performance with step size learning



Up to 3x faster than Gurobi

OSQP features

Features

Robust

Embeddable
(can be division free!)

Supports
warm-starting

Detects
infeasibility

Can improve with
data

OSQP features

Features

Robust

Embeddable
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Can improve with
data

Learning for OSQP

Integrate RL and ADMM to dynamically
tune parameters

- Faster convergence
- Very low overhead
- Interpretable policy

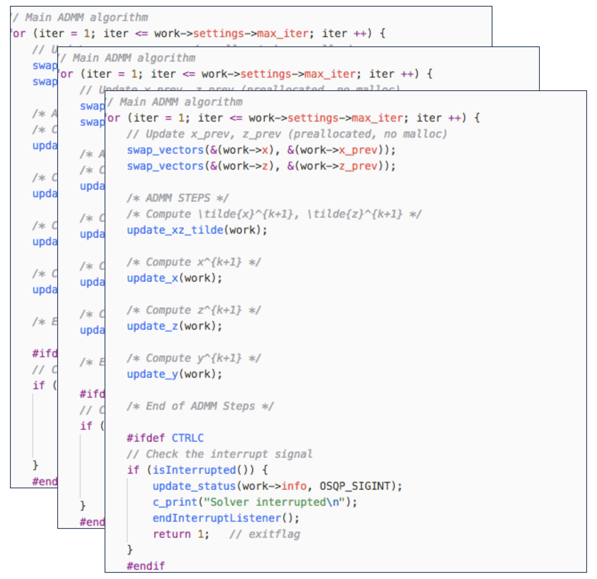
OSQP 1.0 (this summer!)

Improved embedded code generation

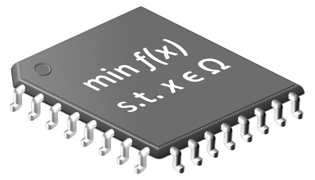
```
# Create OSQP object
m = osqp.OSQP()

# Initialize solver
m.setup(P, q, A, l, u,
        settings)

# Generate C code
m.codegen('folder_name')
```



Embedded Hardware



- Code generation in C
- CVXPY integration

OSQP 1.0 (this summer!)

Improved embedded code generation

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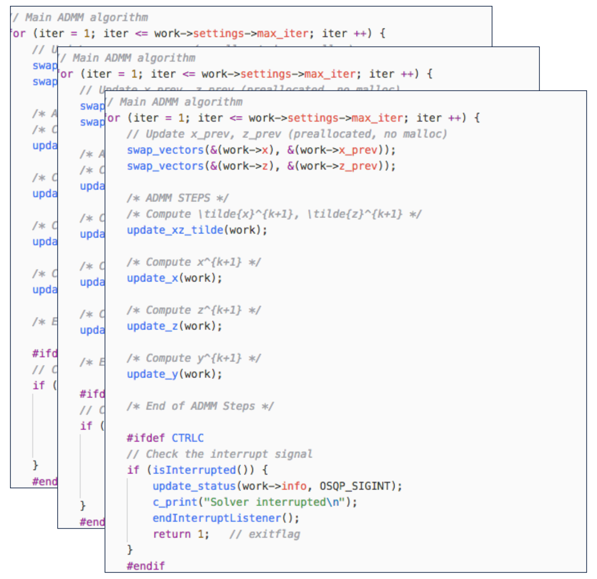

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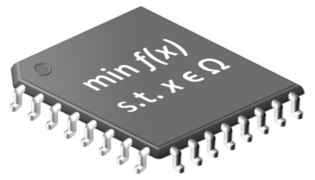

- Code generation in C
- CVXPY integration

Differentiable layers

PyTorch

OSQP 1.0 (this summer!)

Improved embedded code generation

```python
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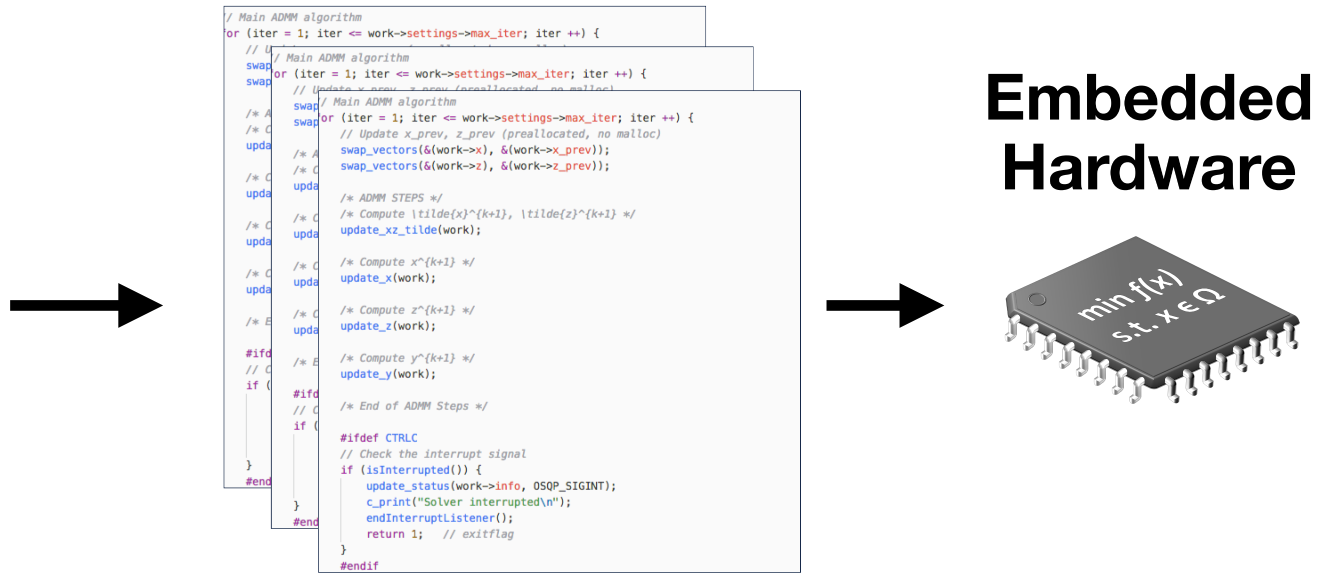


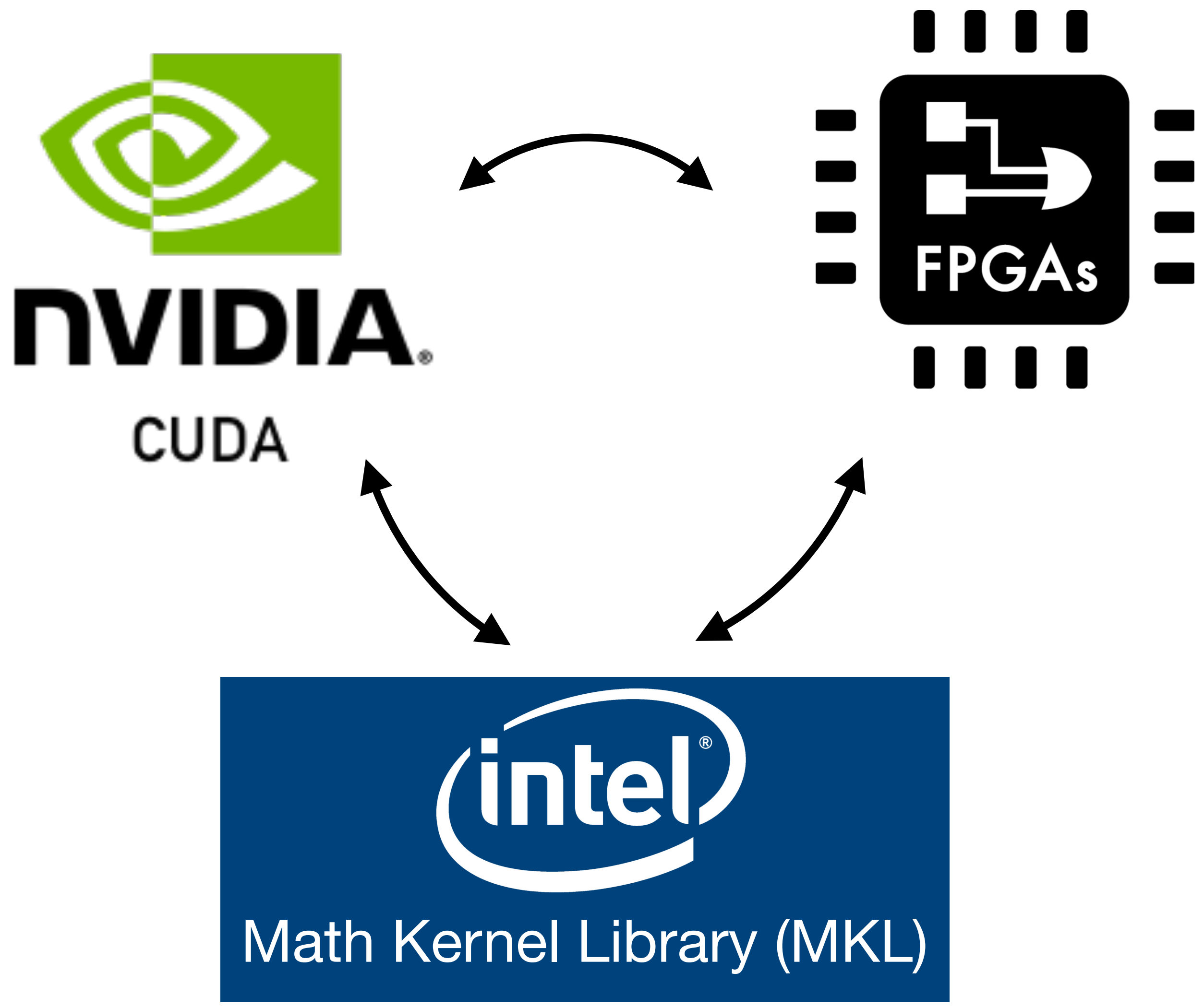
- Code generation in C
- CVXPY integration

Differentiable layers

JAX

PyTorch

Modular linear algebra

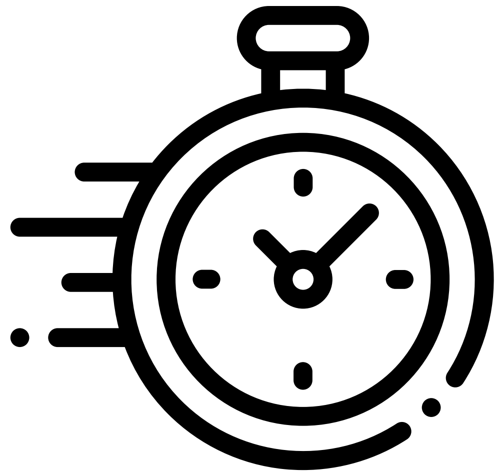


Today’s talk

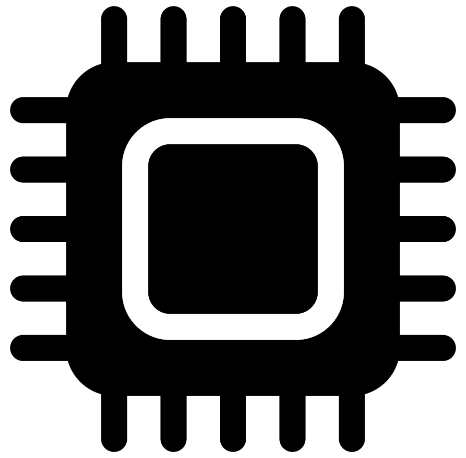
Data-Driven Embedded Optimization for Control

OSQP Solver

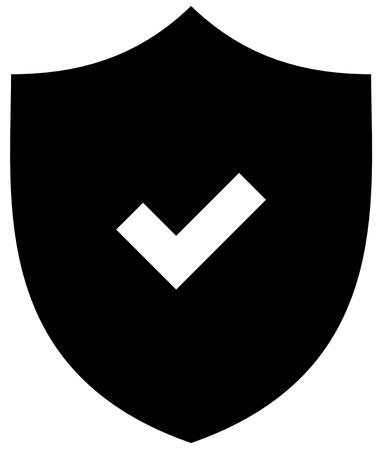
Real-Time



Limited resources

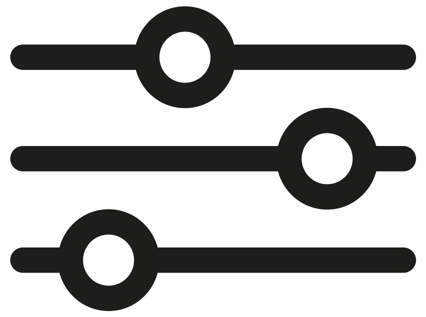


Reliability



Learning Convex Optimization Control Policies

Interpretable tuning



Today's talk

Data-Driven Embedded Optimization for Control

OSQP Solver

Real-Time

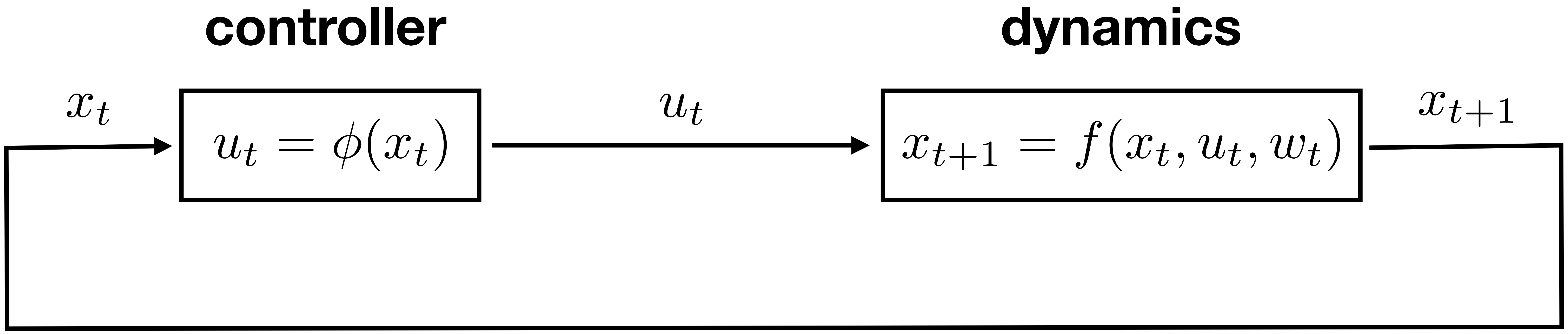
Limited resources

Reliability

Learning Convex Optimization Control Policies

Interpretable tuning

Control loop



x_t state
u_t input
w_t (random) disturbance

$\phi(x_t)$ **control policy**

Explicit vs implicit control policies

Explicit

Complete control specification

Explicit vs implicit control policies

Explicit

Complete control specification

Example: PI Controller

$$u_t = -K_P e_t - K_I \sum_{\tau=0}^{t} e_\tau$$

Explicit vs implicit control policies

Explicit

Complete control specification

Example: PI Controller

$$u_t = -K_P e_t - K_I \sum_{\tau=0}^{t} e_\tau$$

Implicit (optimization-based)

Designer specifies goal and requirements $\longrightarrow$ **Optimizer** computes the action

Explicit vs implicit control policies

Explicit

Complete control specification

Example: PI Controller

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Designer specifies goal and requirements $\longrightarrow$ **Optimizer** computes the action

Example: LQR Controller

dynamics: $x_{k+1} = Ax_t + Bu_t + w_t$
stage cost: $x^T Q x + u^T R u$

Explicit vs implicit control policies

Explicit

Complete control specification

Example: PI Controller

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Designer specifies goal and requirements $\longrightarrow$ **Optimizer** computes the action

Example: LQR Controller

dynamics: $x_{k+1} = Ax_t + Bu_t + w_t$
stage cost: $x^T Q x + u^T R u$

$\downarrow$

$$\begin{aligned} u_t &= \underset{u}{\text{argmin}}\, u^T R u + (Ax_t + Bu)^T P (Ax_t + Bu) \\ &= K x_t \end{aligned}$$

Convex optimization control policies (COCPs)

$$
\begin{array}{rll}
u_t = & \underset{u}{\text{argmin}} & f(x_t, u, \theta) \\
& \text{subject to} & g(x_t, u, \theta) \leq 0 \\
& & A(x_t, \theta)u = b(x_t, \theta)
\end{array}
$$

x_t state
θ parameters to tune
f, g convex functions

Convex optimization control policies (COCPs)

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Many control policies are COCPs

Examples

- Linear Quadratic Regulator (LQR)
- Model predictive control (MPC)
- Actuator allocation
- Resource allocation
- Portfolio trading

Many control policies are COCPs

Examples

- Linear Quadratic Regulator (LQR)
- Model predictive control (MPC)
- Actuator allocation
- Resource allocation
- Portfolio trading

Advantages

Interpretable

Satisfy constraints

Handle varying dynamics

Efficient and reliable
(even division-free: OSQP)

Judging COCPs

Given a policy, state and input trajectories form a ***stochastic process***

Judging COCPs

Given a policy, state and input trajectories form a ***stochastic process***

Trajectories

$$X = (x_0, \ldots, x_{T-1}, x_T)$$
$$U = (u_0, \ldots, u_{T-1})$$
$$W = (w_0, \ldots, w_{T-1})$$

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Given a policy, state and input trajectories form a ***stochastic process***

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$$\longrightarrow$$

Policy cost

$$J(\theta) = \mathbf{E}\, \psi(X, U, W)$$

Judging COCPs

Given a policy, state and input trajectories form a ***stochastic process***

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$$X = (x_0, \ldots, x_{T-1}, x_T)$$
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$$W = (w_0, \ldots, w_{T-1})$$

$\longrightarrow$

Policy cost

$$J(\theta) = \mathbf{E}\, \psi(X, U, W)$$

Approximate $J(\theta)$ from data (monte carlo simulation)

$$\hat{J}(\theta) = \frac{1}{K} \sum_{i=1}^{K} \psi(X^i, U^i, W^i)$$

COCP Example: dynamic programming

Time-separable cost

$$\psi(X, U, W) = \sum_{t=0}^{T-1} g(x_t, u_t, w_t)$$

COCP Example: dynamic programming

Time-separable cost

$$\psi(X, U, W) = \sum_{t=0}^{T-1} g(x_t, u_t, w_t)$$

Optimal policy as $T \to \infty$

$$\phi(x_t) = \underset{u}{\text{argmin}}\, \mathbf{E}\left(g(x_t, u, w_t) + V\left(f(x_t, u, w_t)\right)\right)$$

COCP Example: dynamic programming

Time-separable cost

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Value function

COCP Example: dynamic programming

Time-separable cost

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Optimal policy as $T \to \infty$

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Value function

COCP if

- f affine in x and u
- g convex in x and u
- V is convex

COCP Example: approximate dynamic programming

$$\phi(x_t) = \underset{u}{\text{argmin}}\, \mathbf{E}\left(g(x_t, u, w_t) + \hat{V}(f(x_t, u, w_t))\right)$$

COCP Example: approximate dynamic programming

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Approximate value function

COCP Example: approximate dynamic programming

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Approximate value function

COCP if

- f affine in x and u
- g convex in x and u
- $\hat{V}$ is convex

(even when V is not)

Controller tuning problem

Goal

$$\text{minimize} \quad J(\theta)$$

Nonconvex and difficult to solve

Controller tuning problem

Goal

$$\text{minimize} \quad J(\theta)$$

Nonconvex and difficult to solve

Traditional approaches

- **Hand-tuning** (few parameters, simple dependencies)
- **Derivative-free method** (very slow)

Learning scheme

Auto-tuning

Stochastic gradient descent

$$\theta^{k+1} = \theta^k - t^k \nabla_\theta \hat{J}(\theta^k)$$

Learning scheme

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Stochastic gradient descent

$$\theta^{k+1} = \theta^k - t^k \nabla_\theta \hat{J}(\theta^k)$$

step size

Learning scheme

Auto-tuning

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$$\theta^{k+1} = \theta^k - \underbrace{t^k}_{\text{step size}} \underbrace{\nabla_\theta \hat{J}(\theta^k)}_{\text{stochastic gradient from simulation}}$$

step size

stochastic gradient from simulation

Learning scheme

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step size

stochastic gradient
from simulation

Generalization

Split simulation data in
training, validation and testing

Learning scheme

Auto-tuning

Stochastic gradient descent

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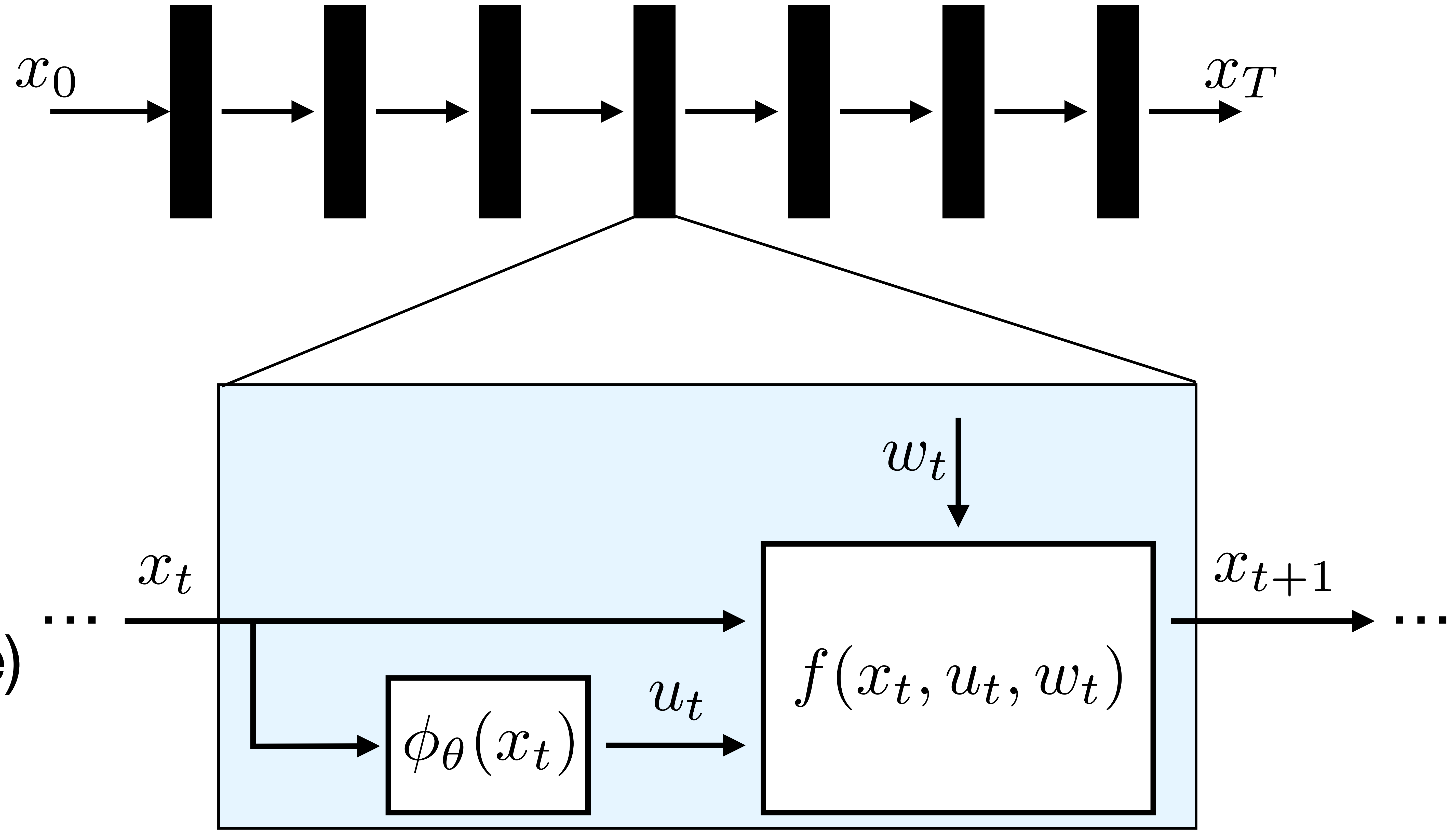
Generalization

Split simulation data in training, validation and testing

Non differentiable $\hat{J}(\theta)$?

Still get a descent direction (common in NN community)

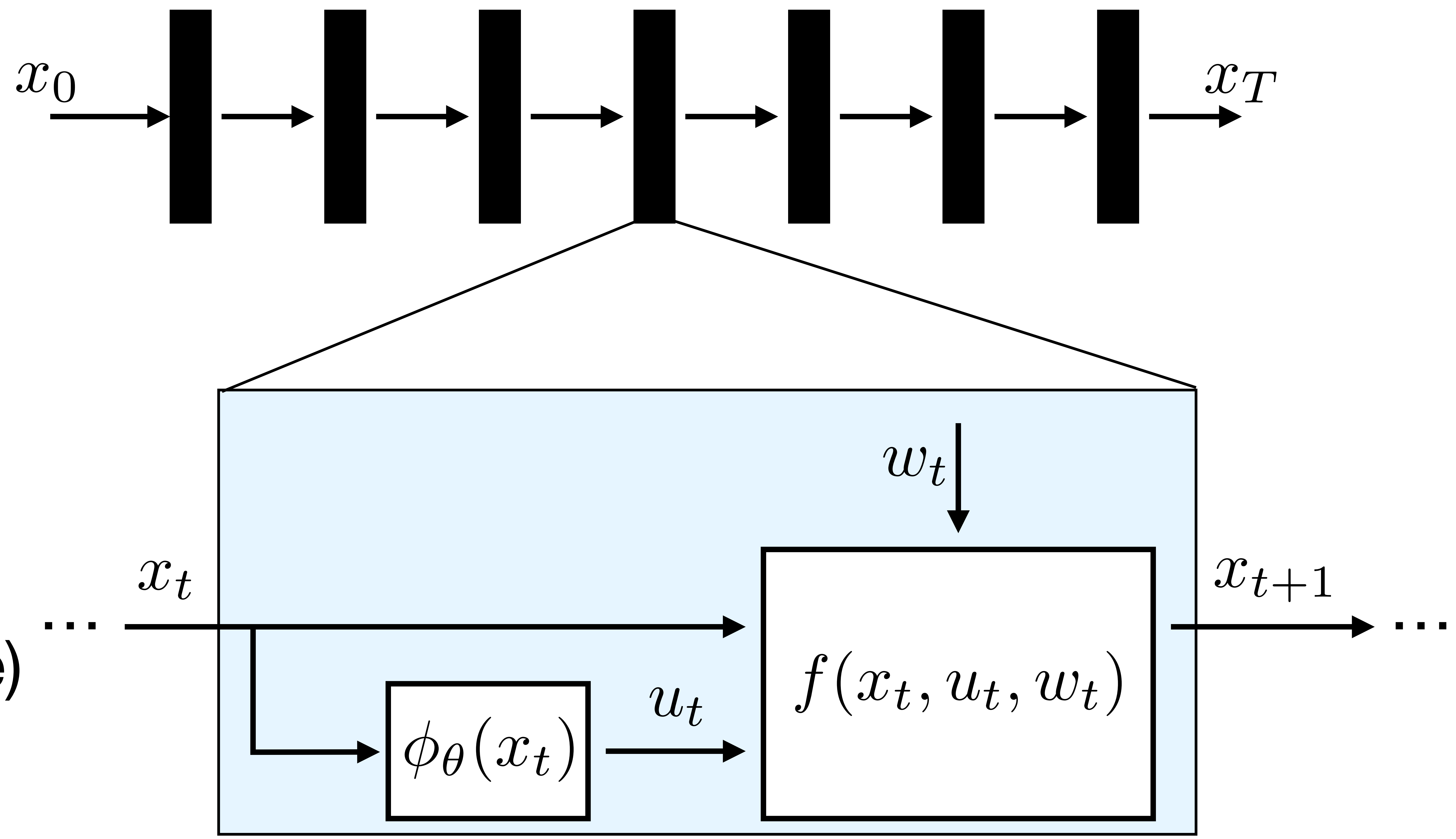
Implementation



Automatic differentiation

- **Build** computation graph (simulate)
- **Backpropagate** using PyTorch

Implementation



Automatic differentiation

- **Build** computation graph (simulate)
- **Backpropagate** using PyTorch

CVXPYLayers

Backpropagate through COCPs
(differentiate KKT optimality conditions)

[https://github.com/cvxgrp/cvxpylayers/]

[Differentiable convex optimization layers. Agrawal, Amos, Barratt, Boyd, Diamond, and Kolter. NeurIPS 2019]

[Differentiable optimization-based modeling for machine learning. Amos. PhD thesis 2019]

Box-constrained LQR

Problem setup

- dynamics: $x_{t+1} = Ax_t + Bu_t + w_t$
- actuator limit: $\|u_t\|_\infty \leq 1$
- stage cost: $x_t^T Q x_t + u_t^T R u_t$

Box-constrained LQR

Problem setup

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COCP Policy (QP)

$$u_t = \begin{array}{ll} \underset{u}{\text{argmin}} & u^T R u + \|\theta(Ax_t + Bu)\|_2^2 \\ \text{subject to} & \|u\|_\infty \leq 1 \end{array}$$

Box-constrained LQR

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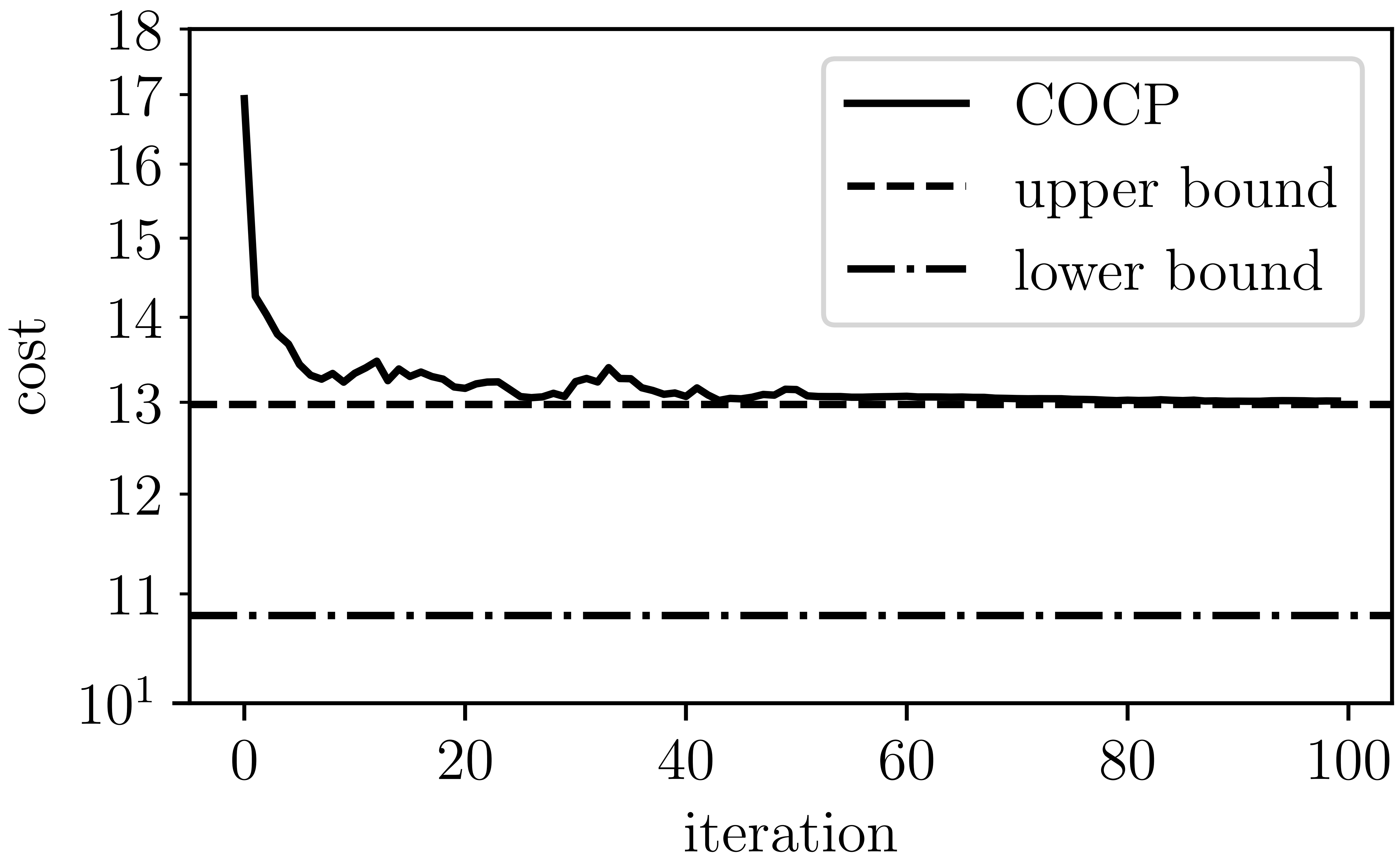
parameters

Box-constrained LQR

Performance

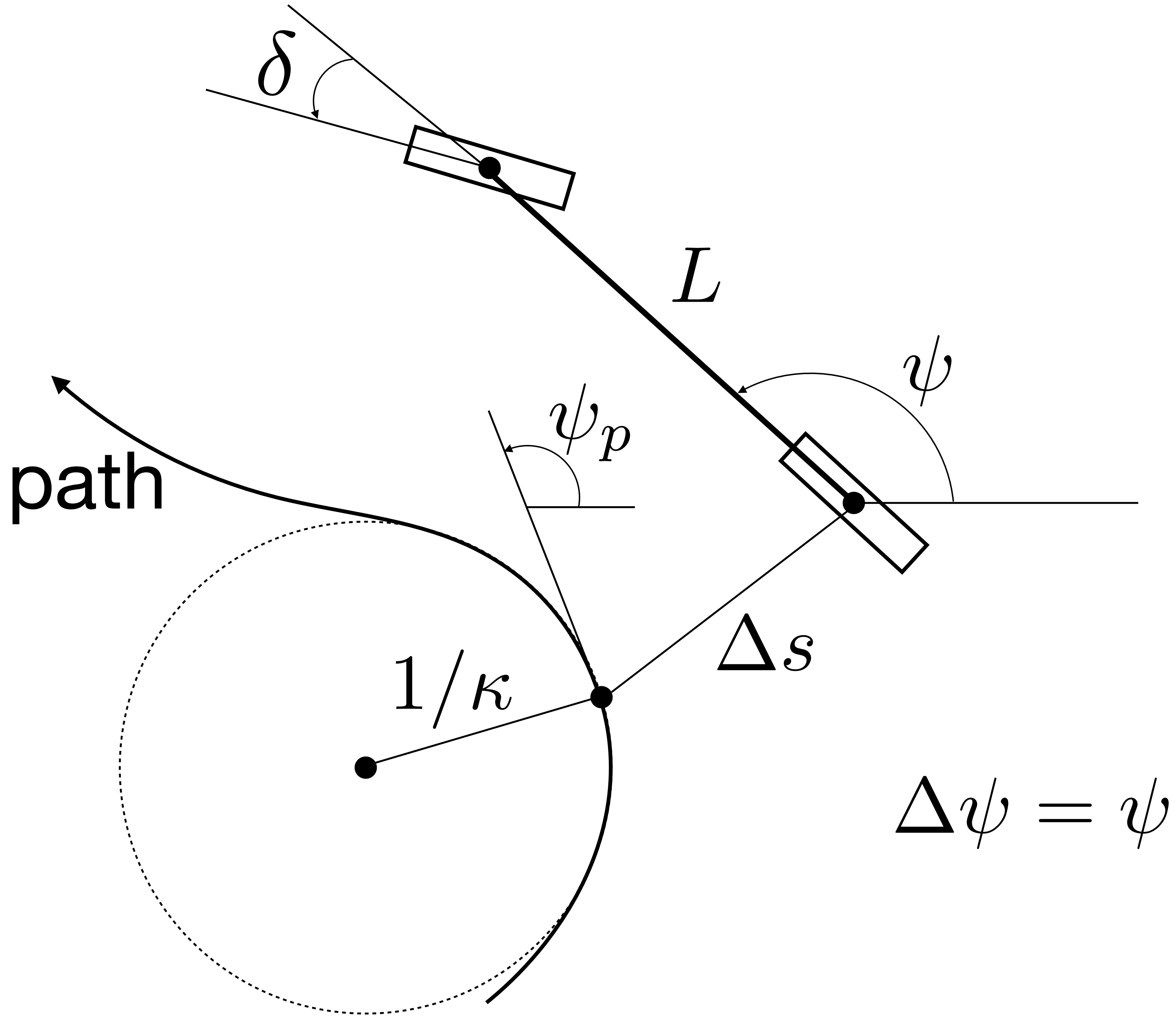


Standard upper/lower bounds from SDPs

Hard to generalize
(other dynamics, disturbances, etc)

Vehicle tracking curved paths

State: $x_t = (\Delta s_t, \Delta\psi_t, v_t, v_t^{\mathrm{des}}, \kappa_t)$

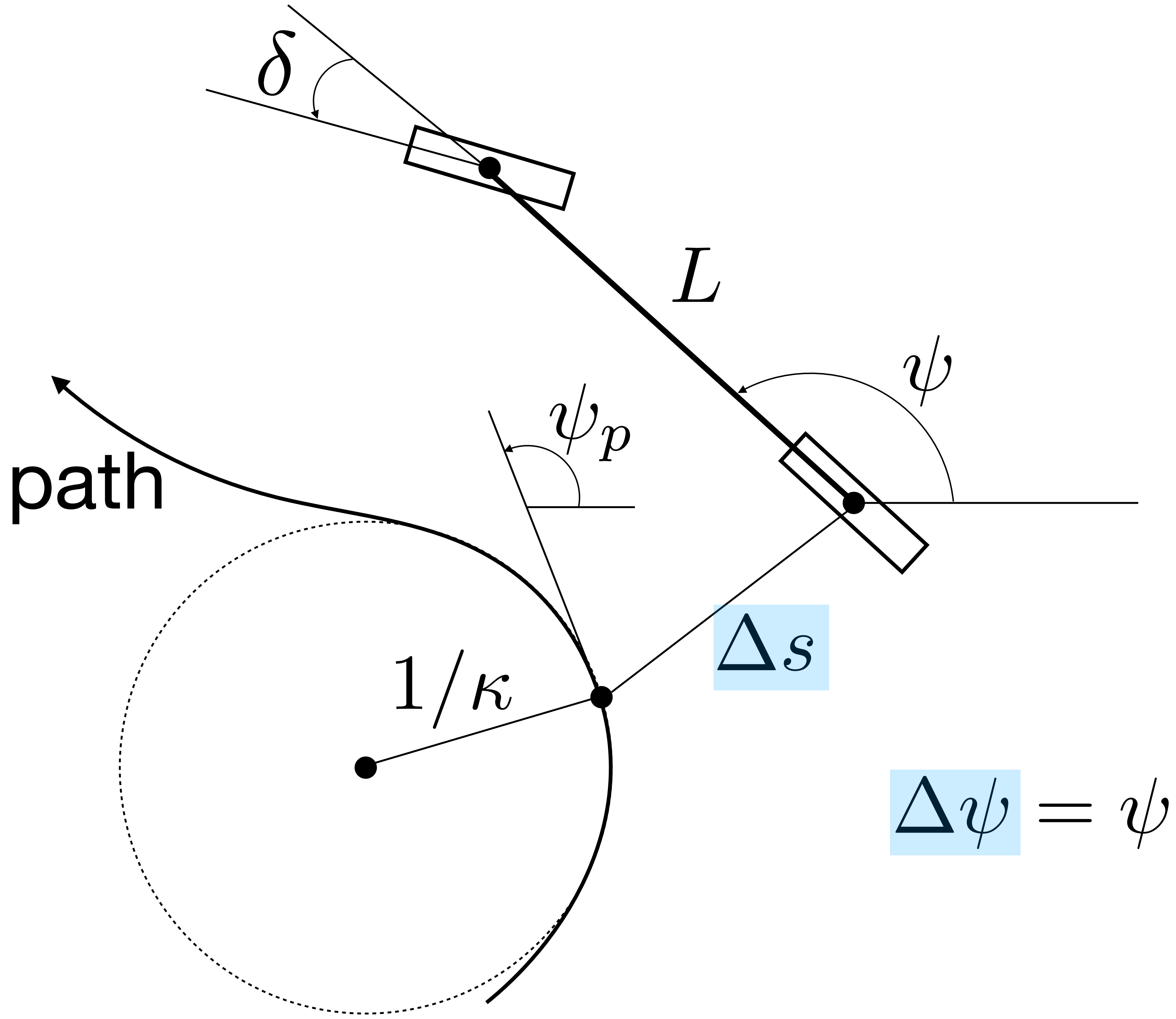


$\Delta\psi = \psi - \psi_p$

Vehicle tracking curved paths

lateral deviation   heading deviation

State: $x_t = (\Delta s_t, \Delta\psi_t, v_t, v_t^{\mathrm{des}}, \kappa_t)$



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lateral deviation heading deviation

path curvature

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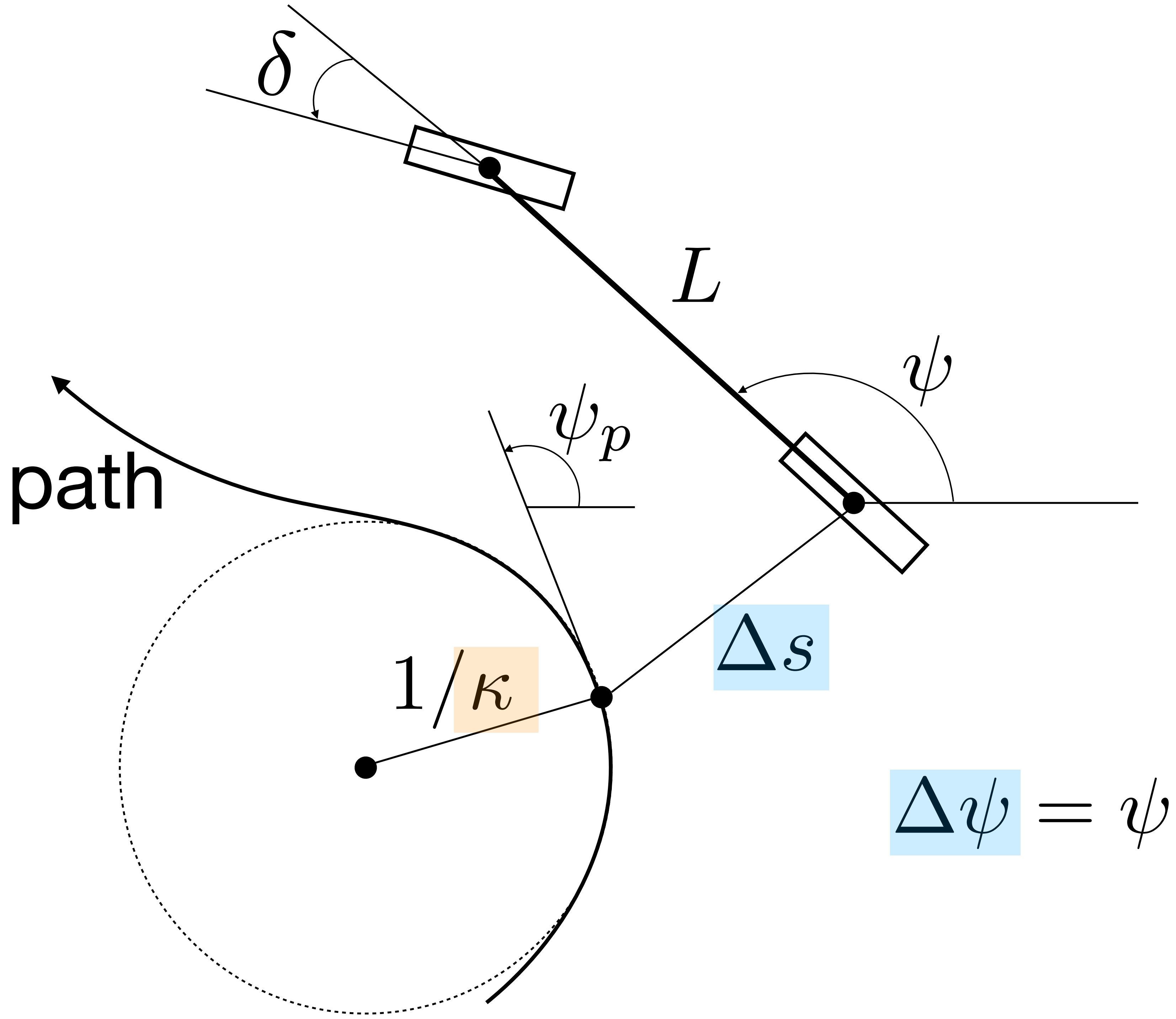

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Vehicle tracking curved paths

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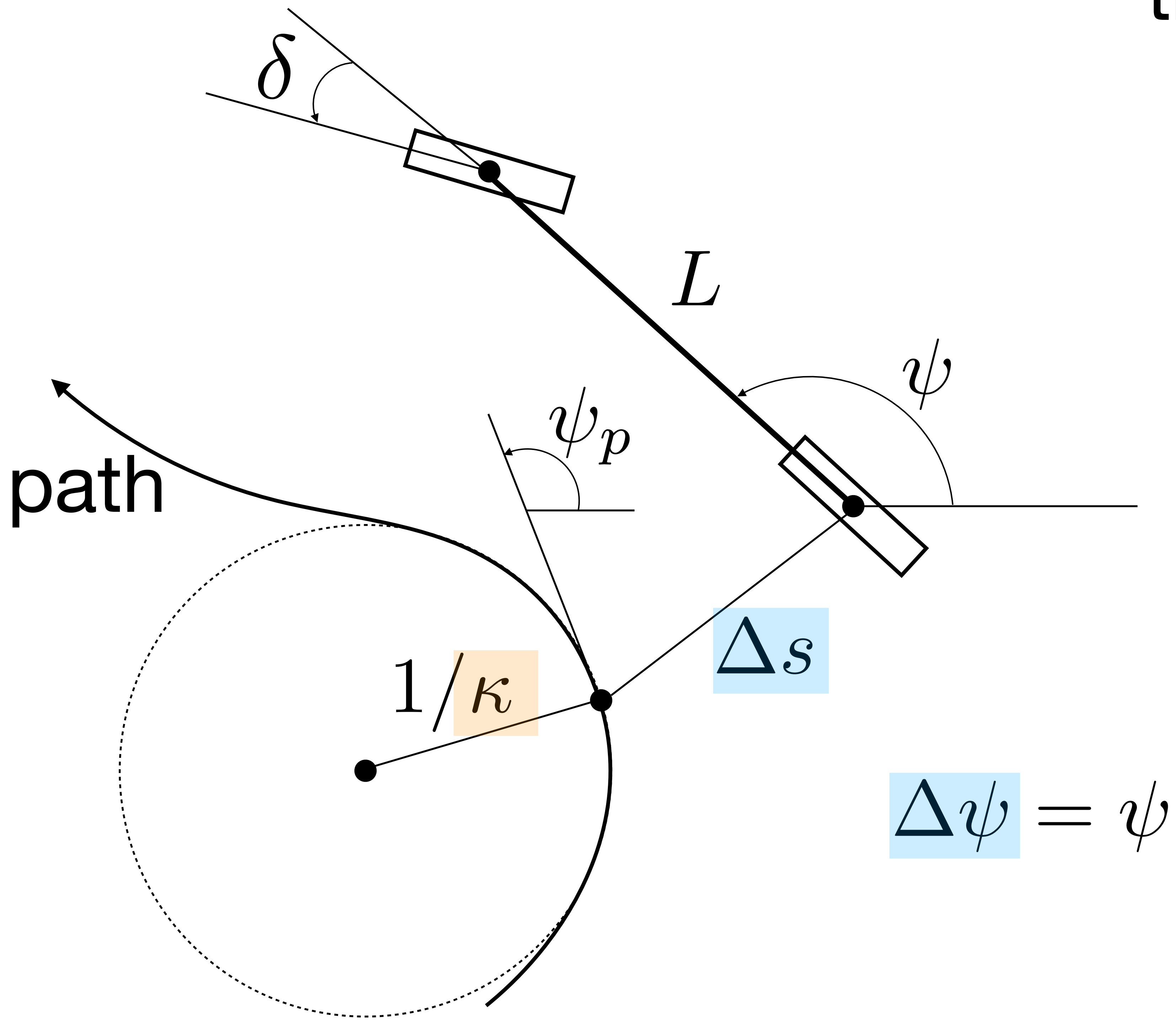
State: $x_t = (\Delta s_t, \Delta\psi_t, v_t, v_t^{\mathrm{des}}, \kappa_t)$

true and desired velocities



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Vehicle tracking curved paths

lateral deviation   heading deviation

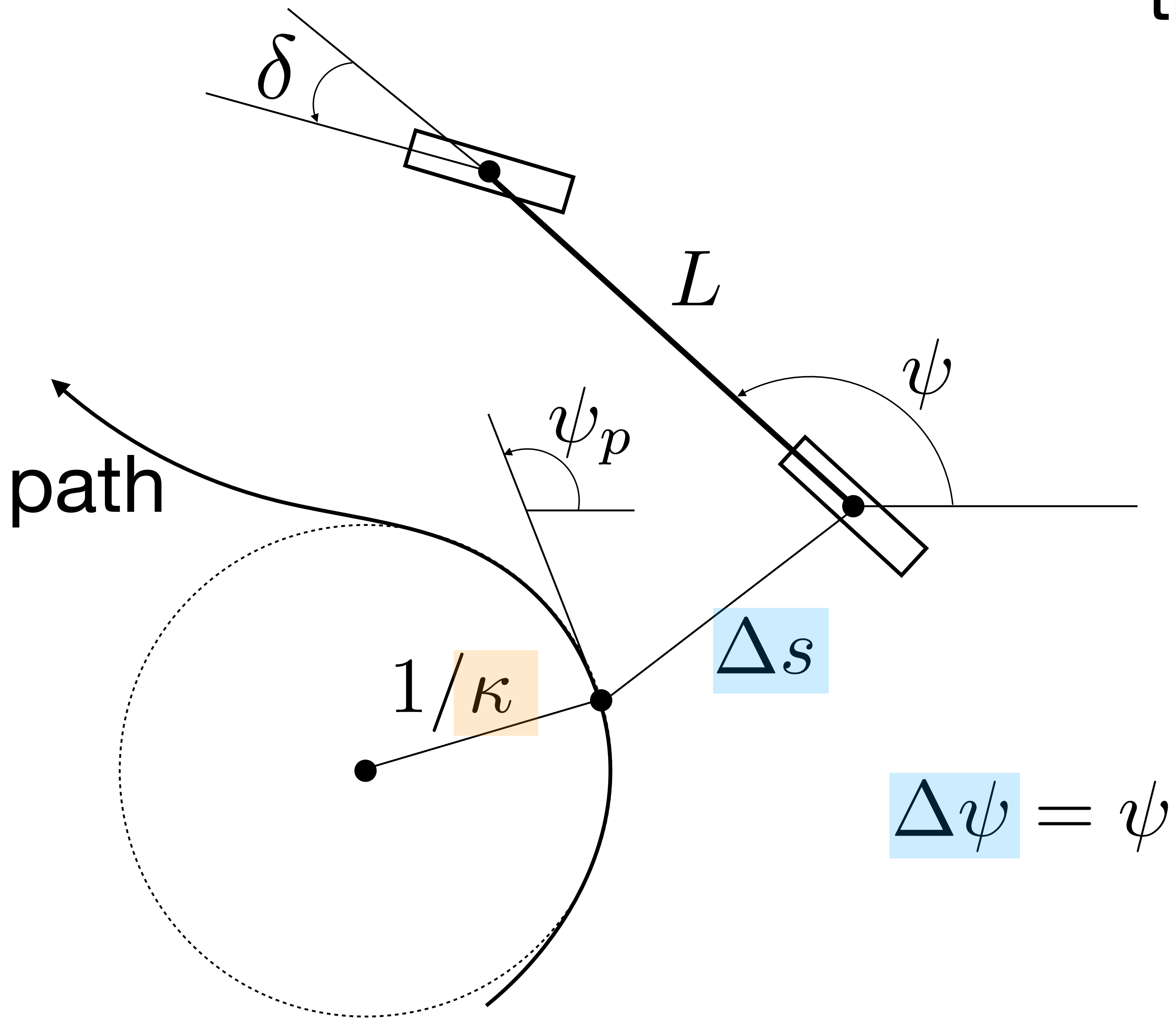
path curvature

State: $x_t = (\Delta s_t, \Delta\psi_t, v_t, v_t^{\mathrm{des}}, \kappa_t)$

true and desired velocities

acceleration

Input: $u_t = (a_t, z_t)$



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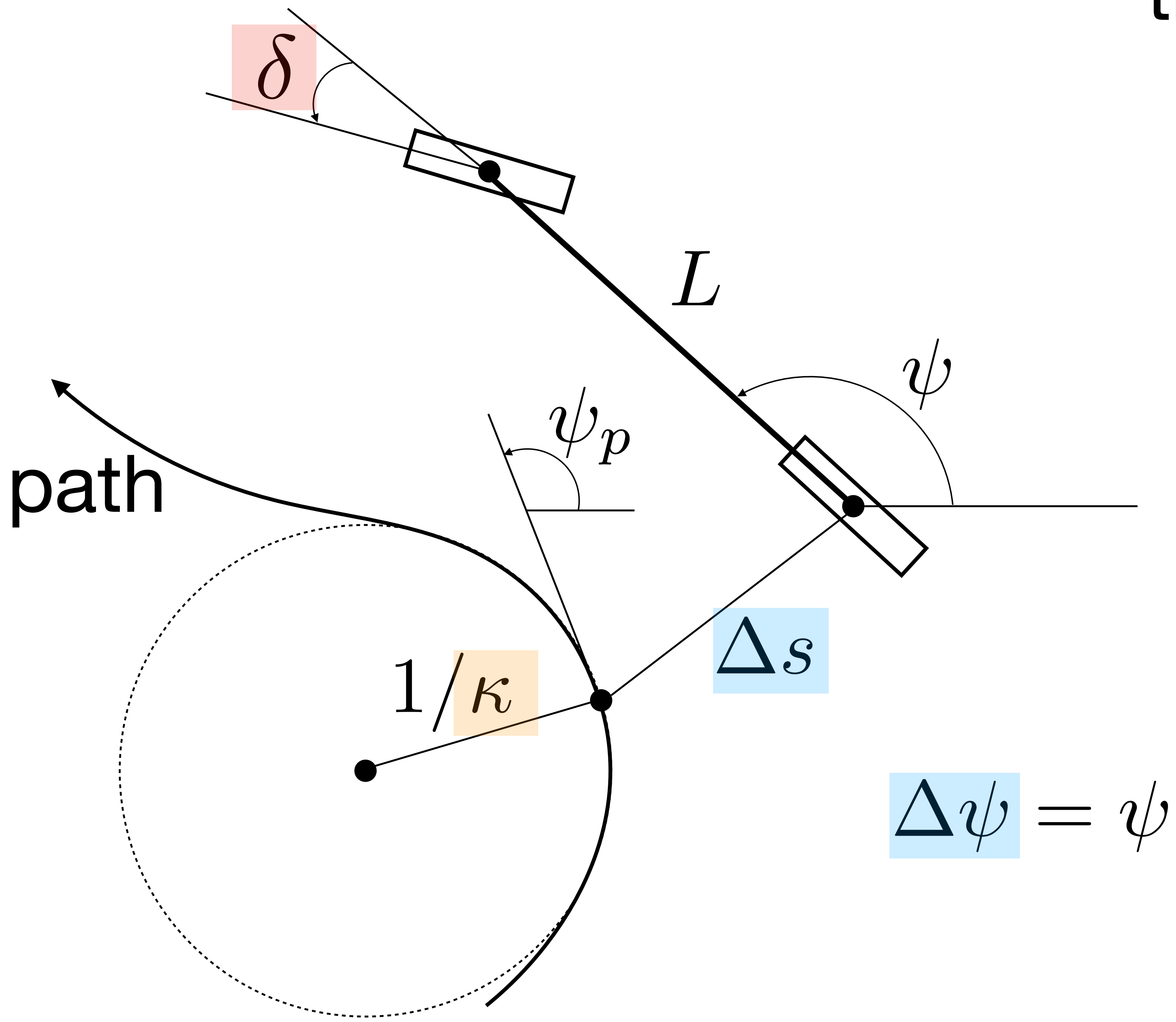
acceleration

Input: $u_t = (a_t, z_t)$

turn

$$z_t = \tan(\delta_t) - L\kappa_t$$



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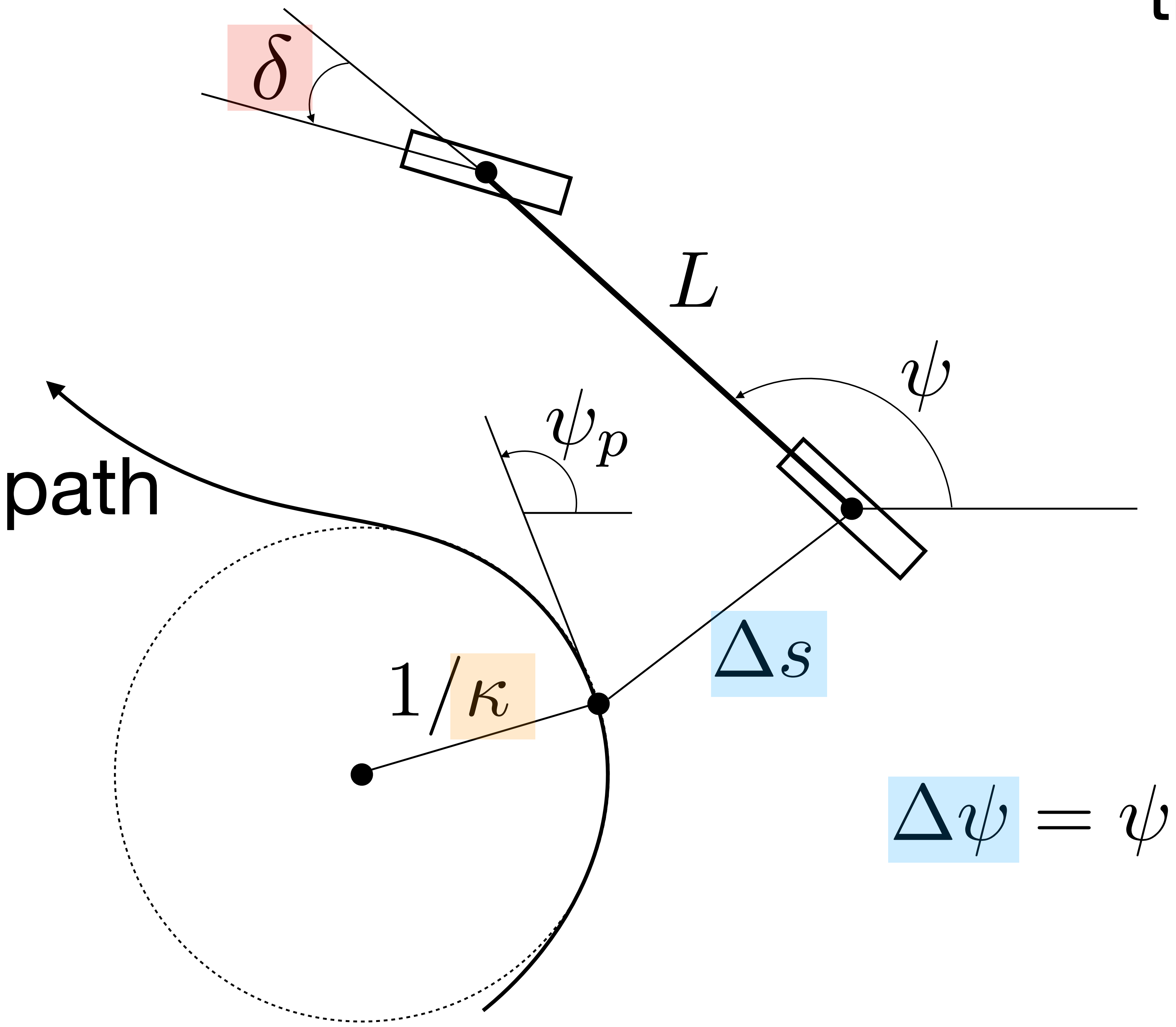

$$\Delta\psi = \psi - \psi_p$$

Dynamics

$$x_{t+1} = f(x_t, u_t, w_t)$$

Vehicle tracking curved paths

Cost and constraints

Stage cost

$$(v_t - v_t^{\mathrm{des}})^2 + \Delta s_t^2 + \Delta \psi_t^2 + \lambda(|a_t| + z_t^2)$$

Vehicle tracking curved paths

Cost and constraints

track velocity

Stage cost

$$\boxed{(v_t - v_t^{\text{des}})^2} + \Delta s_t^2 + \Delta \psi_t^2 + \lambda(|a_t| + z_t^2)$$

Vehicle tracking curved paths

Cost and constraints

track velocity

Stage cost

$$\underbrace{(v_t - v_t^{\text{des}})^2}_{\text{track velocity}} + \underbrace{\Delta s_t^2 + \Delta \psi_t^2}_{\text{track path}} + \lambda(|a_t| + z_t^2)$$

track path

Vehicle tracking curved paths

Cost and constraints

Stage cost

track velocity

small control effort

$$\boxed{(v_t - v_t^{\text{des}})^2} + \boxed{\Delta s_t^2 + \Delta \psi_t^2} + \lambda\boxed{(|a_t| + z_t^2)}$$

track path

Vehicle tracking curved paths

Cost and constraints

Stage cost

track velocity

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track path

Constraints

maximum acceleration

$$|a_t| \leq a_{\max}$$

maximum turn

$$|z + L\kappa_t| \leq \tan(\delta_{\max})$$

Vehicle tracking curved paths

COCP as a Convex QP

$$
\begin{aligned}
u_t = (a_t, z_t) = \phi(x_t) = \quad \underset{u}{\text{argmin}} \quad & |a| + z^2 + \|Sy\|_2^2 + q^T y \\
\text{subject to} \quad & x_t = (\Delta s_t, \Delta \psi_t, v_t, v_t^{\mathrm{des}}, \kappa_t) \\
& u = (a, z) \\
& y = f(x_t, u, 0) \\
& |a| \leq a_{\max} \\
& |z + L\kappa_t| \leq \tan(\delta_{\max})
\end{aligned}
$$

Vehicle tracking curved paths

COCP as a Convex QP

$$u_t = (a_t, z_t) = \phi(x_t) = \underset{u}{\text{argmin}} \quad |a| + z^2 + \|Sy\|_2^2 + q^T y$$

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$$u = (a, z)$$

next state $\longrightarrow$ $y = f(x_t, u, 0)$

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Vehicle tracking curved paths

COCP as a Convex QP

parameters

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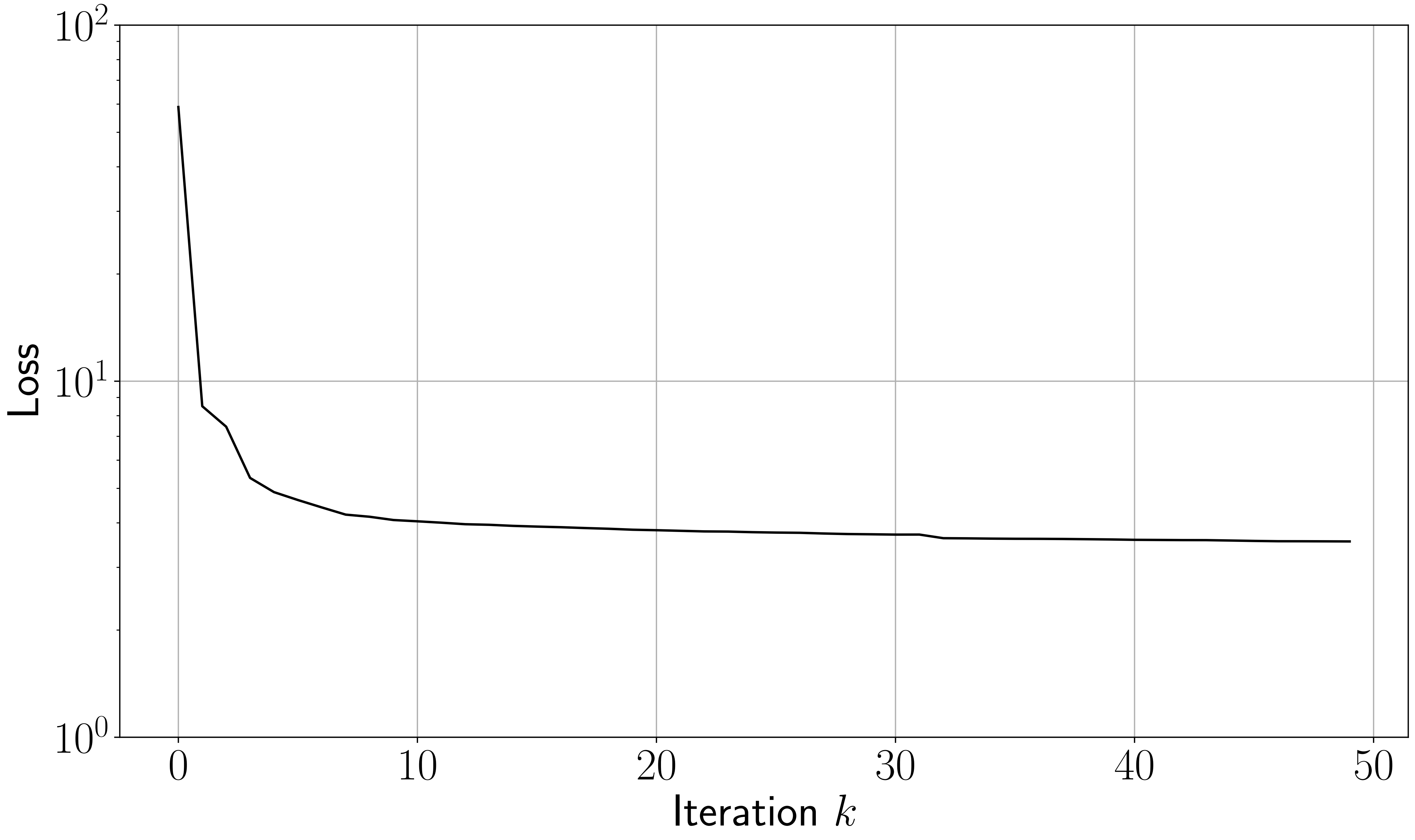
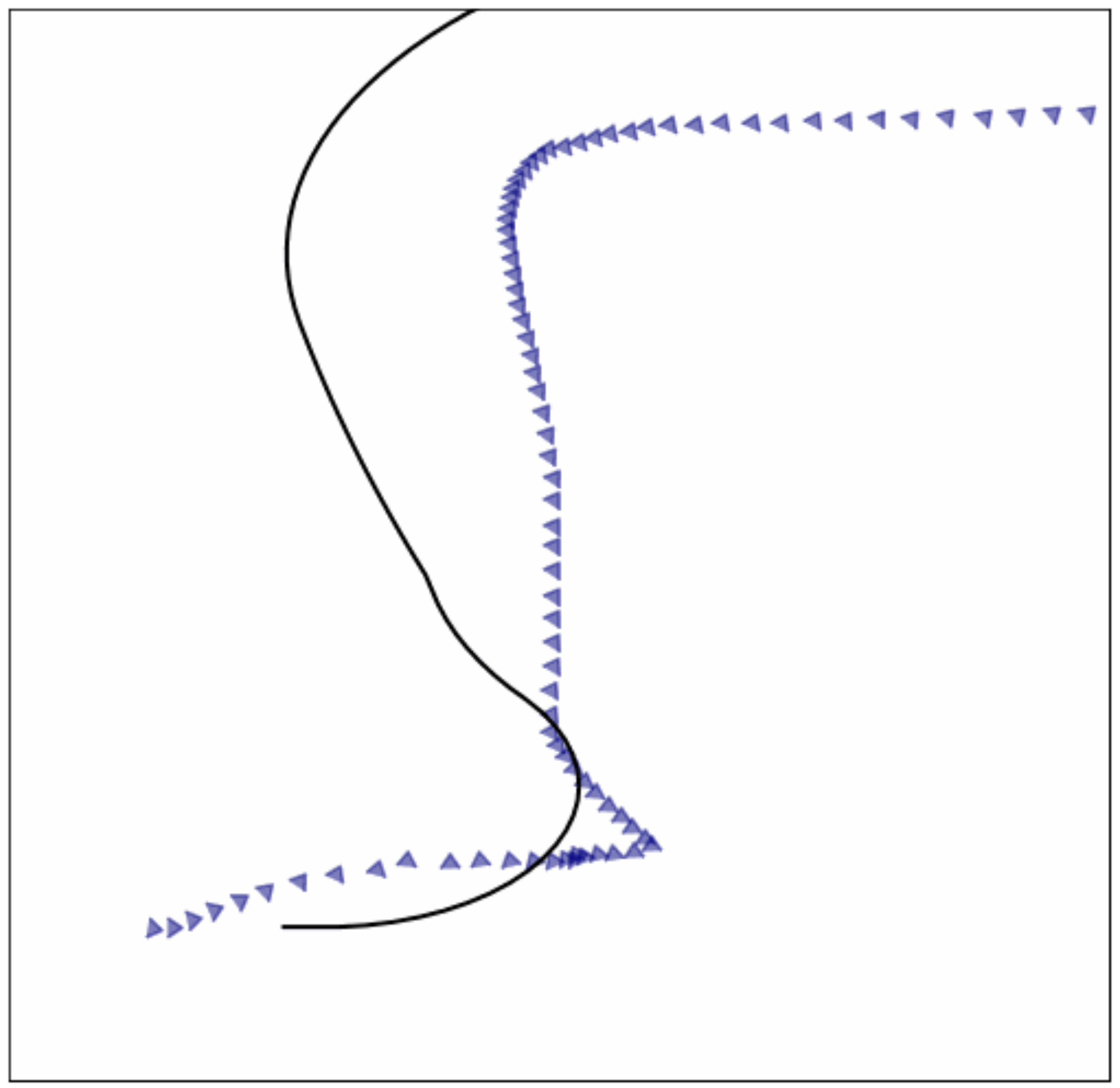
Vehicle tracking curved paths

Results

Validation loss



Tracking behavior

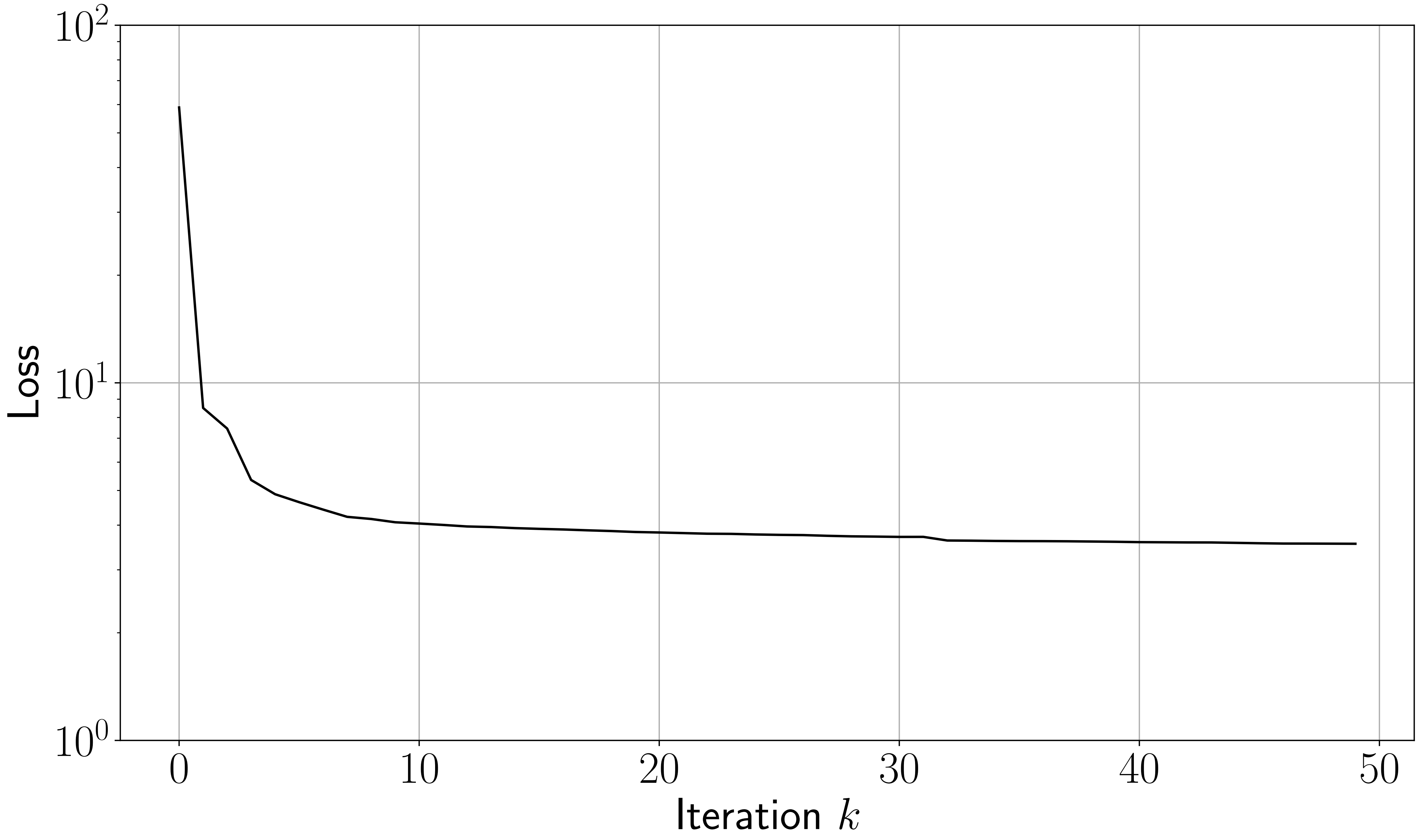


Good trajectory in very few iterations

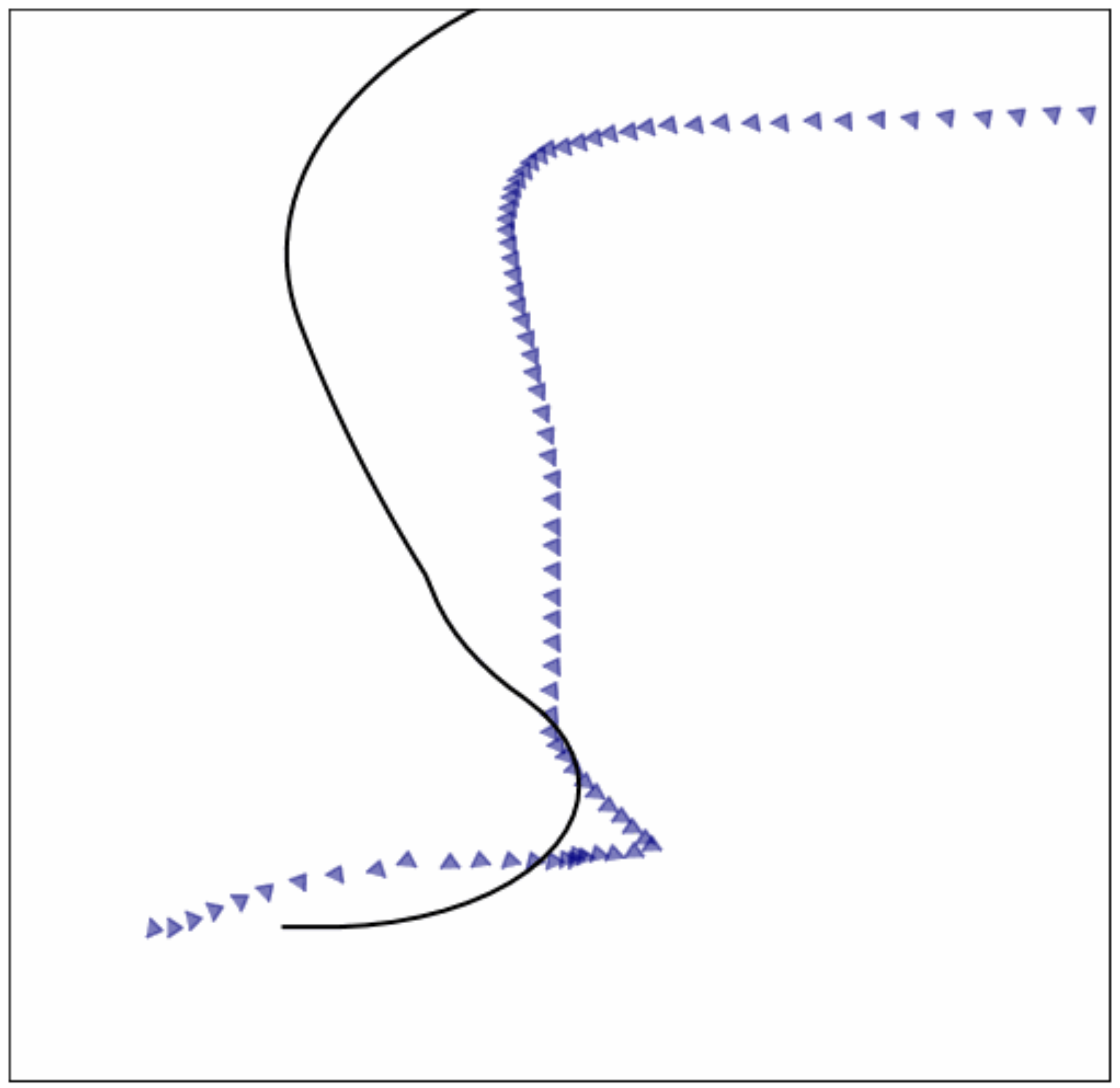
Vehicle tracking curved paths

Results

Validation loss



Tracking behavior



Good trajectory in very few iterations

Learning COCPs summary

Interpretable

Satisfy constraints

Handle varying dynamics

Efficient and reliable (even division-free: OSQP)

Easy to tune from data

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Interpretable

Satisfy constraints

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Easy to tune from data

Future work

- Hybrid (mixed-integer) control policies
- Integrate tuning and deployment with code generation (e.g., OSQP)
- Stochastic policies with safety-guarantees

Conclusions

Acknowledgements

Goran Banjac | Paul Goulart | Alberto Bemporad | Ken Goldberg

   

Stephen Boyd | Akshay Agrawal | Shane Barratt | Jeff Ichnowski

   

Paras Jain | Francesco Borrelli

 

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Conclusions

Real-time and embedded optimization

will soon **be applied everywhere**

Thanks to

Efficient and reliable optimizers

Easy-to-tune control policies

stellato.io

@b_stellato

bstellato@princeton.edu

github.com/bstellato