High-Speed Integer Optimal Control using ADP

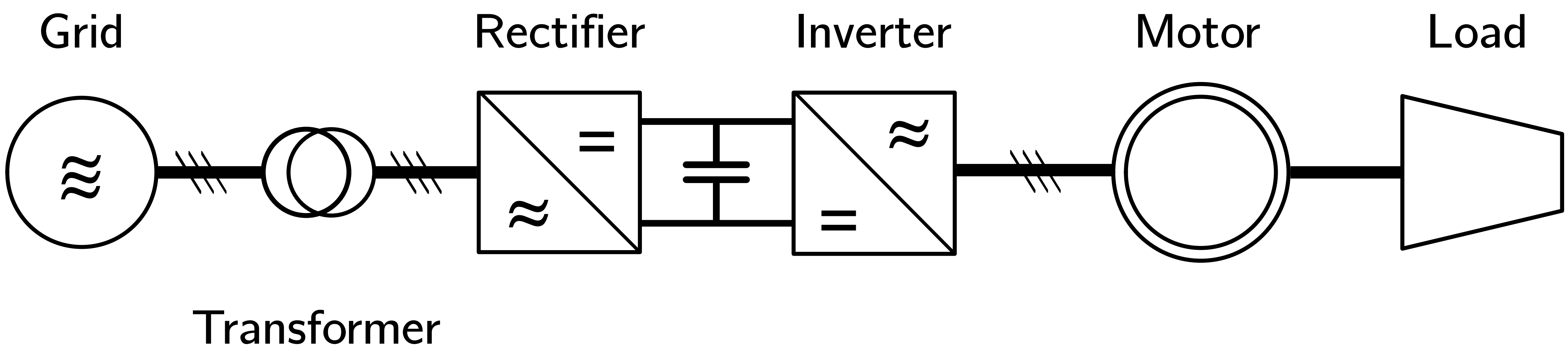
Bartolomeo Stellato and Paul Goulart

UNIVERSITY OF OXFORD

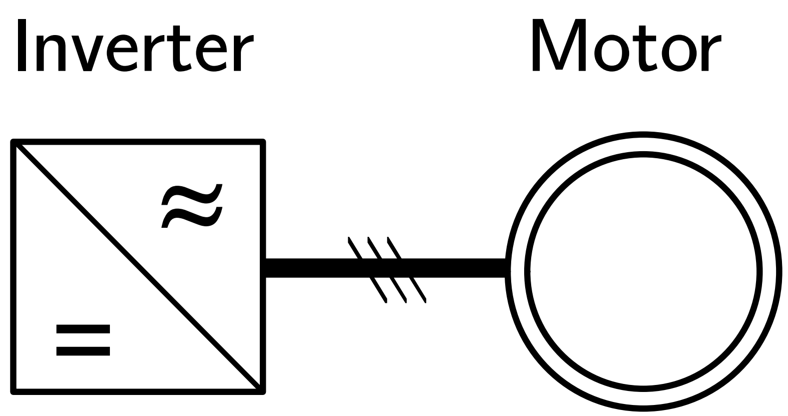
27/10/2016 Berkeley, CA


Medium-voltage drives market is worth 4bln$ ...
... and is growing 10% every year!

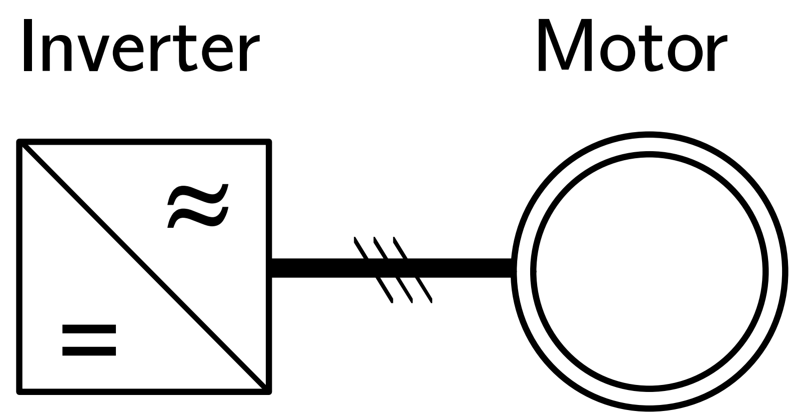
Power Distribution

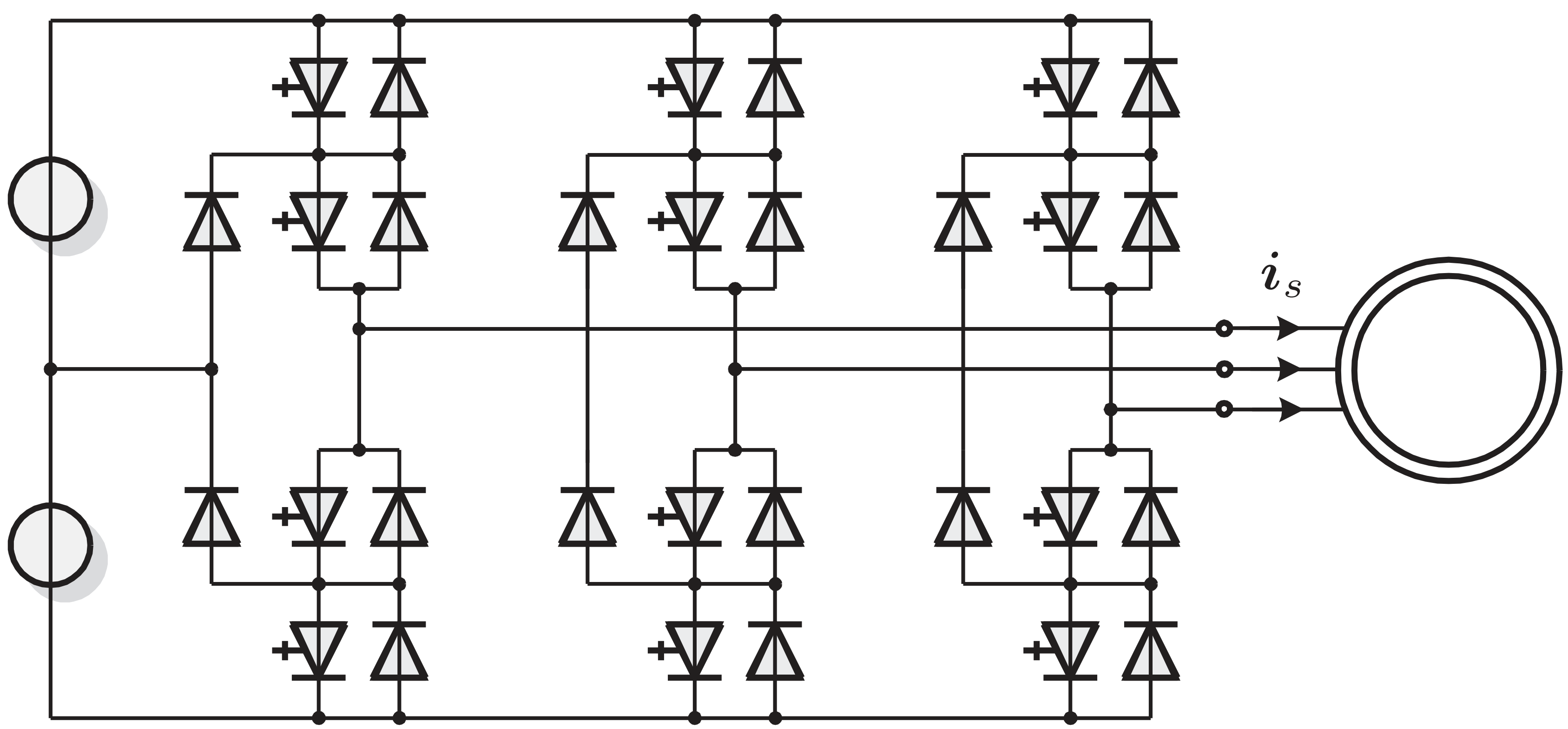


Power Distribution



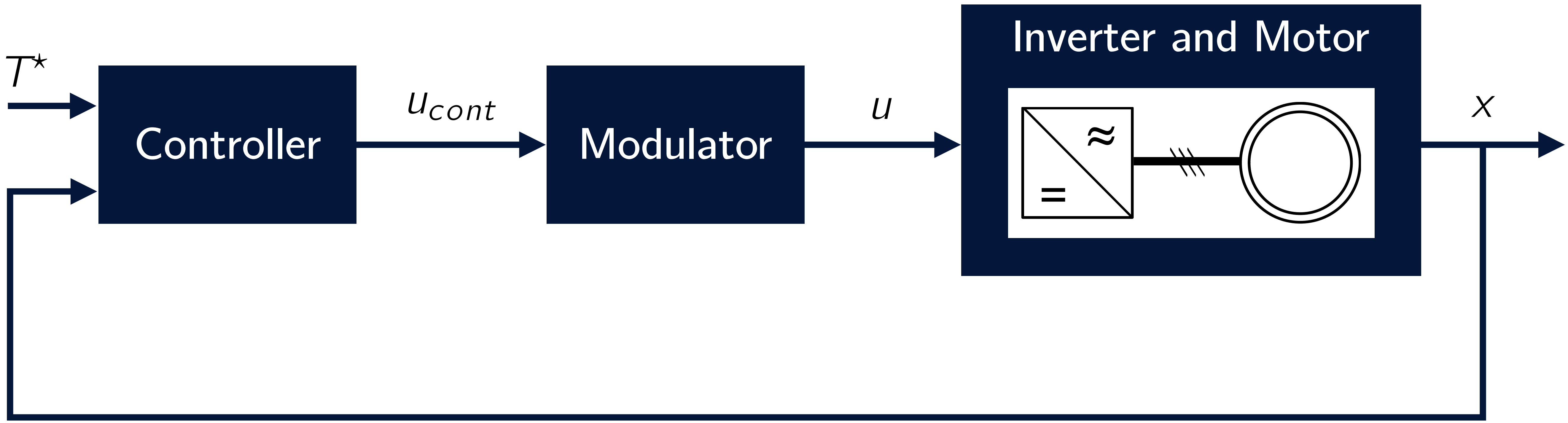
Power Distribution



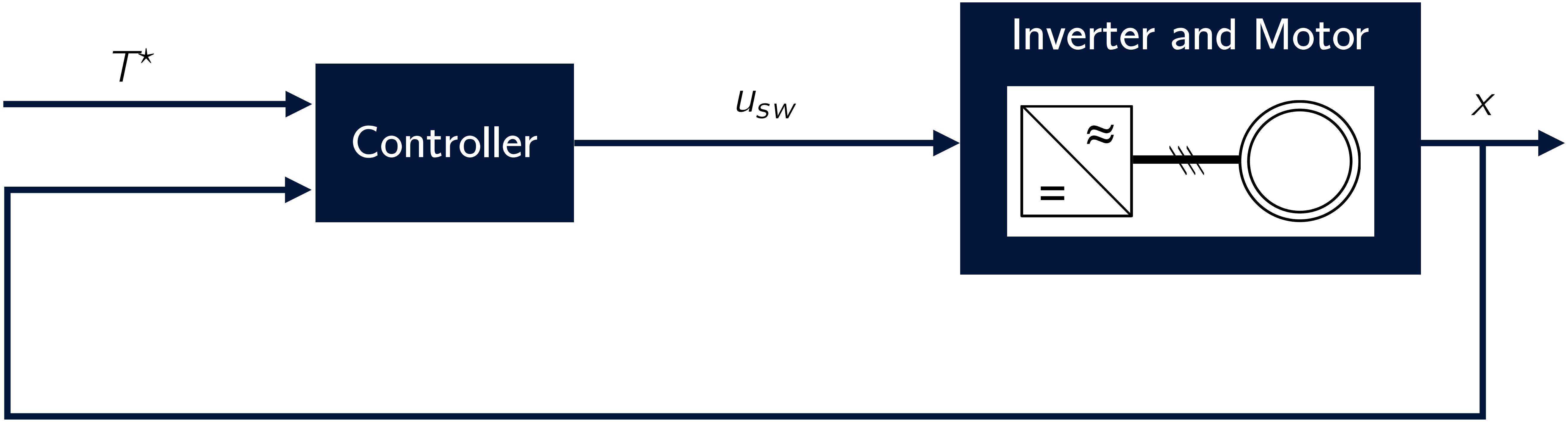


Traditional Control Schemes Are Suboptimal

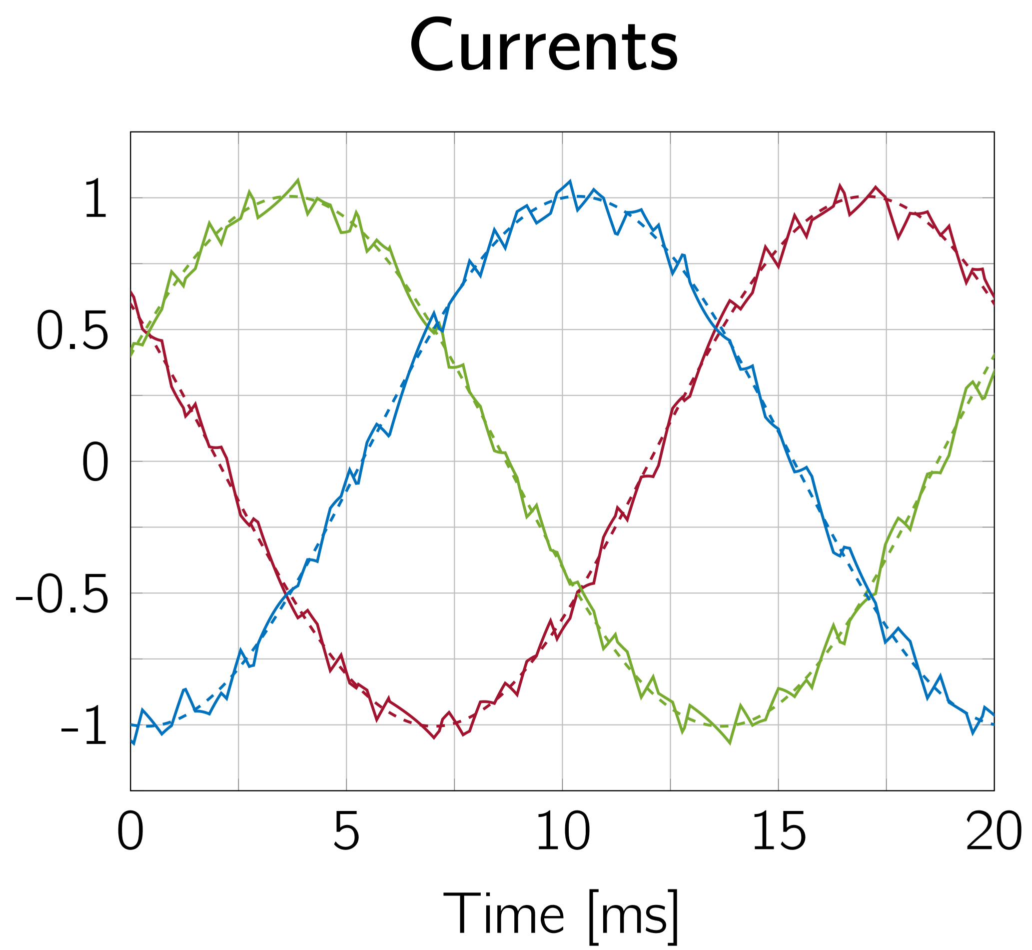
Traditional Control

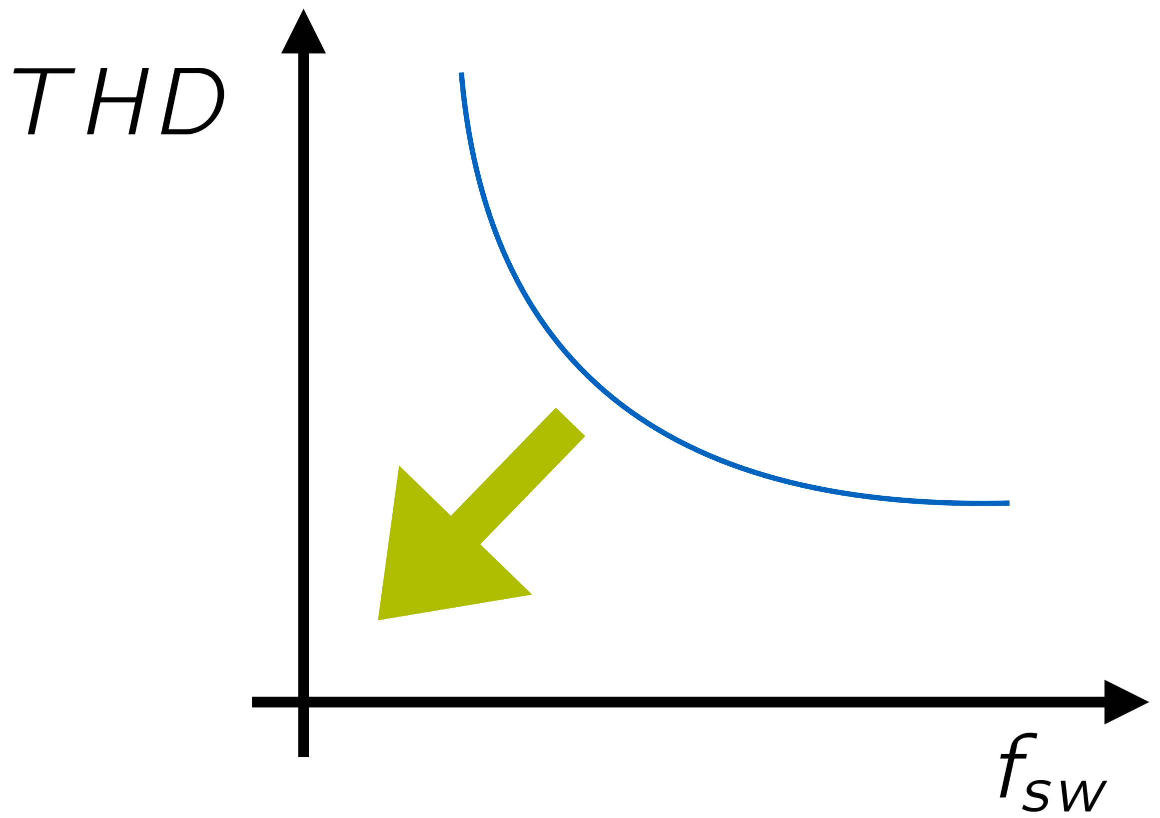


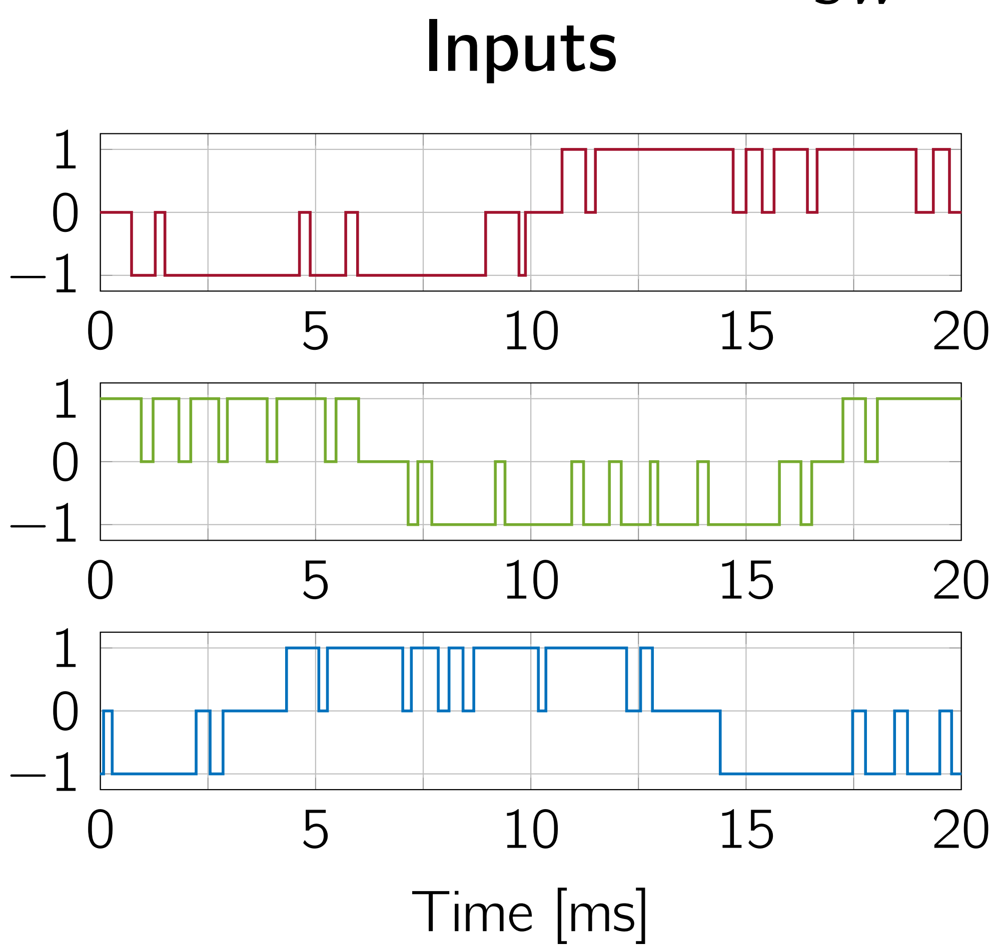
Direct MPC



... But Direct MPC is Difficult

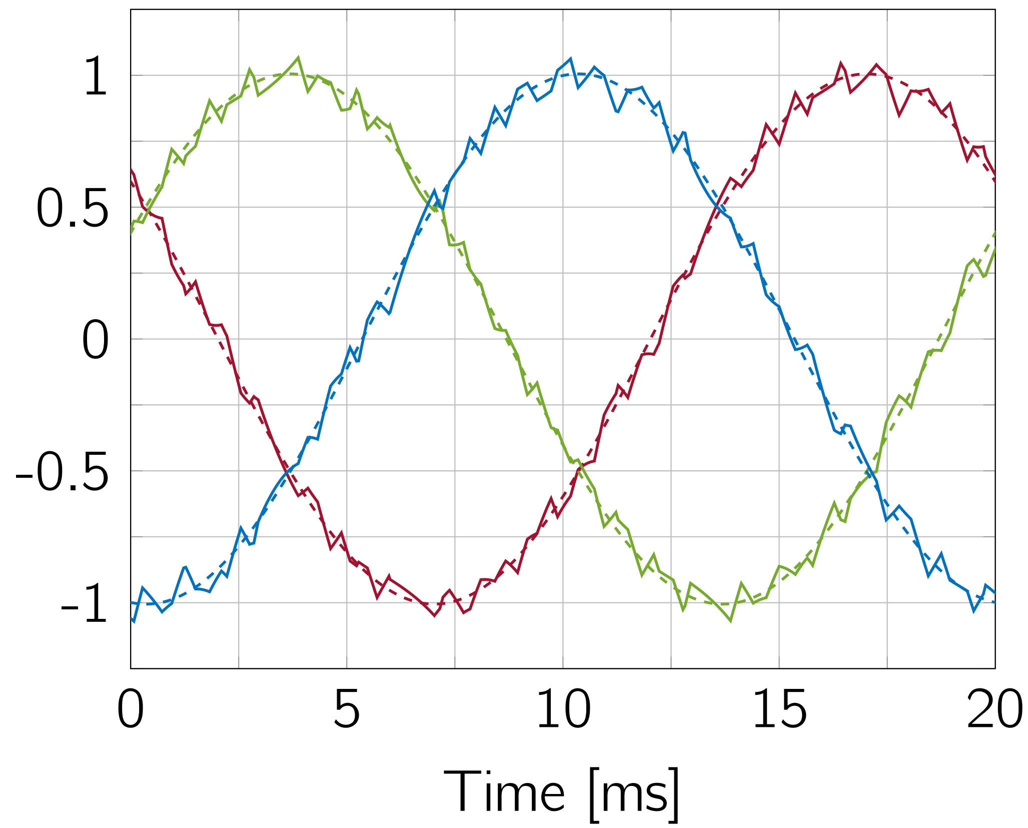


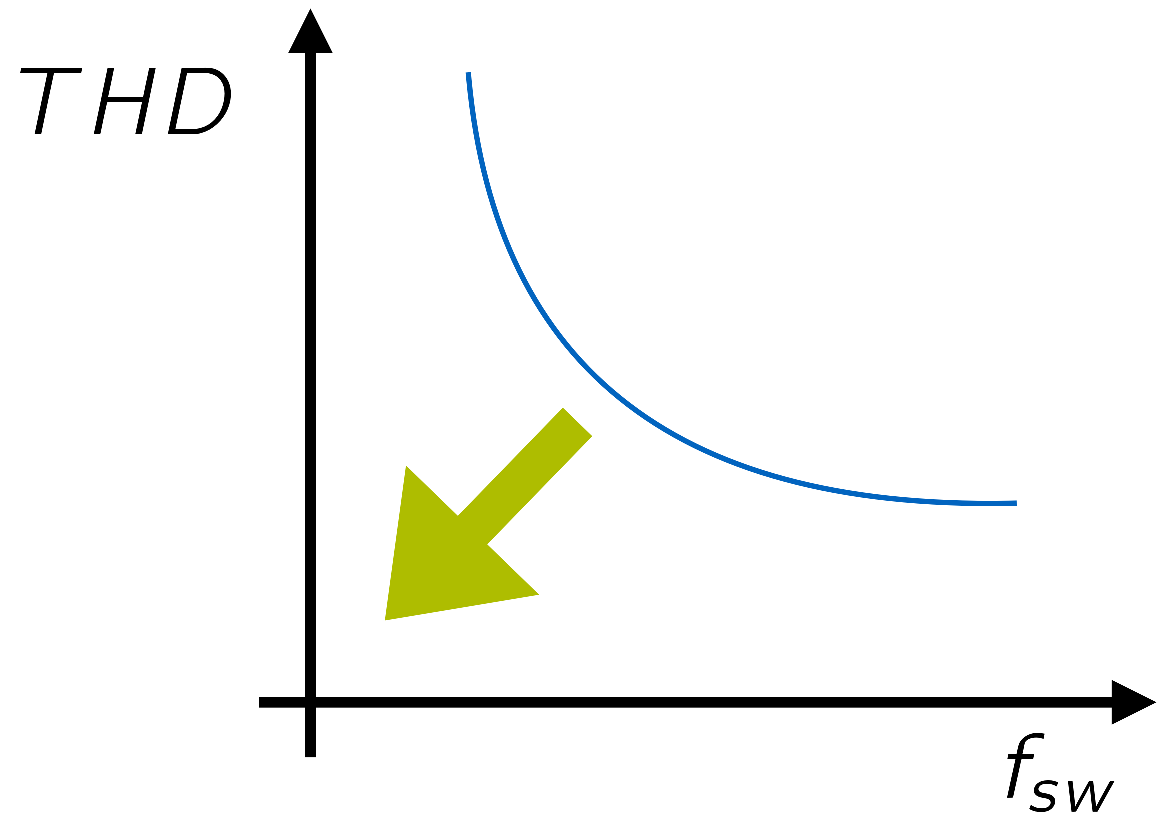




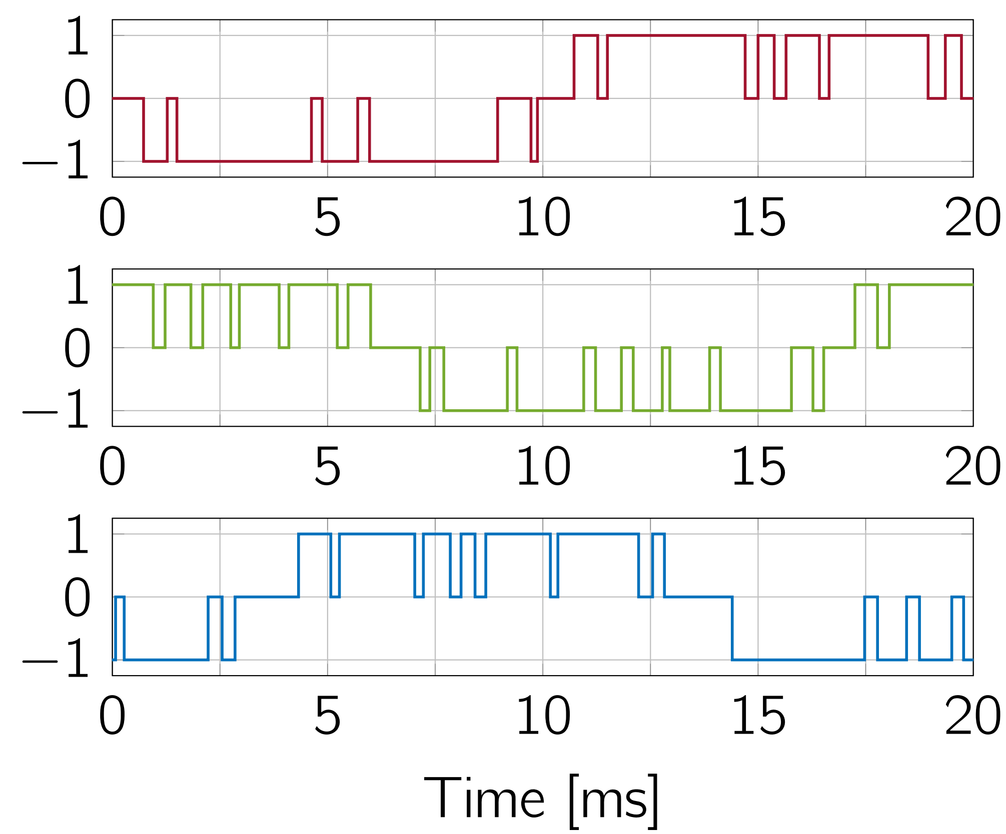
... But Direct MPC is Difficult

Currents





Inputs



Tradeoff

THD **vs** f_{sw}

… But Direct MPC is Difficult

Very fast dynamics

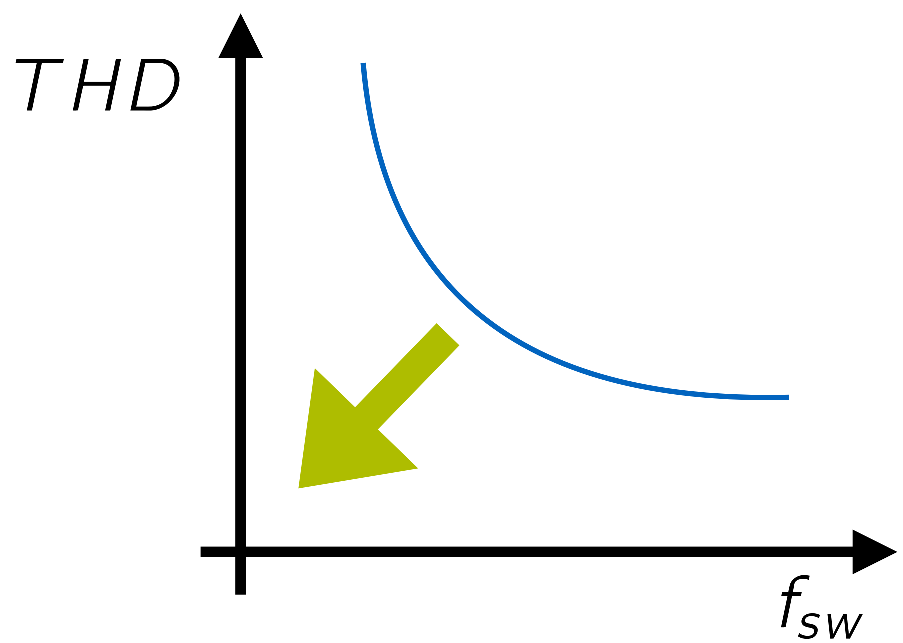
Very fast dynamics

↓

Mixed Integer Optimization Problems in 25 μs!

... But Direct MPC is Difficult

Control Objectives



Tradeoff
THD vs f_{sw}

Timing

Mixed Integer Optimization Problems in 25 μs!

Problem Formulation

Total Harmonic Distortion

$$THD \sim \lim_{M\to\infty} \sqrt{\frac{1}{M} \sum_{k=0}^{M-1} \|i(k) - i^{\star}(k)\|_2^2}$$

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***THD* Component in Cost Function**

$$\sum_{k=0}^{\infty} \gamma^k \|i(k) - i^{\star}(k)\|_2^2$$

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Internal Motor States

x_m

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$$\sum_{k=0}^{\infty} \gamma^k \| i(k) - i^\star(k) \|_2^2$$

Internal Motor States

x_m

Oscillating References

x_{osc}

Switching Frequency

$$f_{sw} = \lim_{T \to \infty} \frac{1}{T} \sum_{k=0}^{T} \|u(k) - u(k-1)\|_1$$

Switching Frequency

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Switching Frequency

University of Oxford

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

$\longrightarrow$ Approximate with IIR Filter

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f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^{\star}\|_2^2$$

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Frequency Estimate

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

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Frequency Estimate

Desired Frequency

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Approximate with IIR Filter

f_{sw} Component in Cost Function

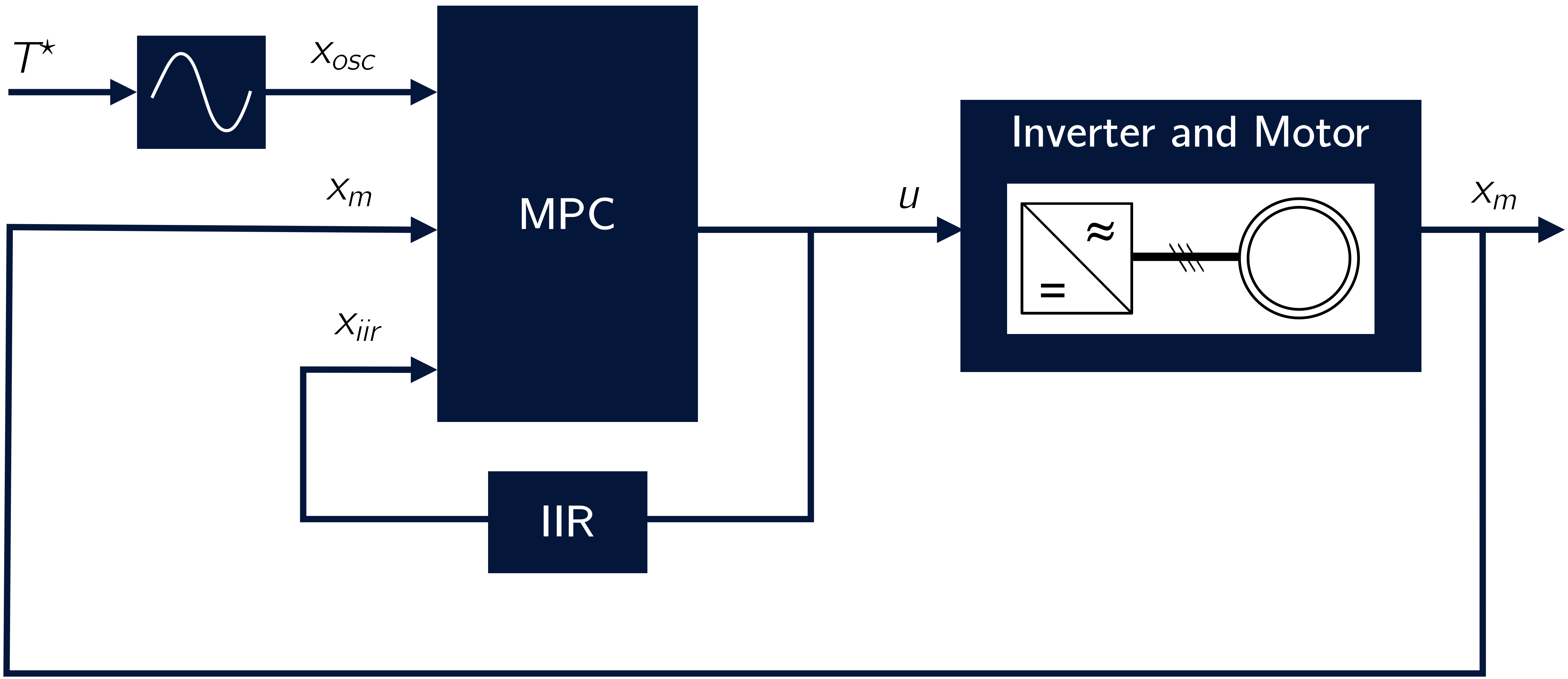
$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f^\star_{sw}\|_2^2$$

Frequency Estimate

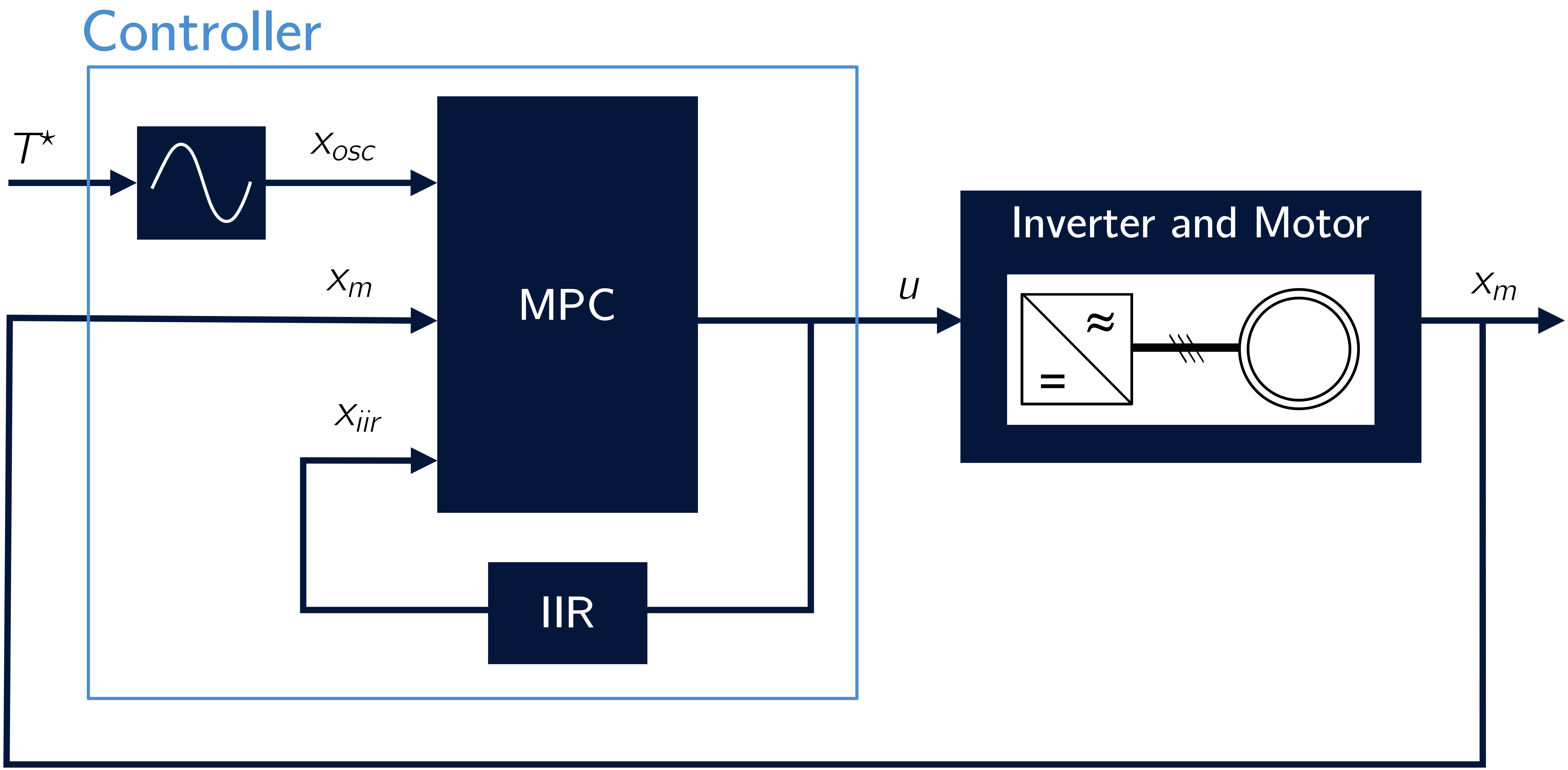
Desired Frequency

Filter States x_{iir}

Complete Block Diagram



States: $x = (x_{osc}, x_{im}, x_{iir})$



States: $x = (x_{osc}, x_{im}, x_{iir})$

Optimal Control Problem

Infinite Horizon

$$
\begin{array}{ll}
\text{minimize} & \displaystyle\sum_{k=0}^{\infty} \gamma^k l(x(k)) \\
\text{subject to} & x(k+1) = Ax(k) + Bu(k) \\
& x(0) = x_0 \\
& x(k) \in \mathcal{X},\ u(k) \in \{-1, 0, 1\}^3
\end{array}
$$

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$$
\text{where} \quad l(x(k)) = \underbrace{\|i(k) - i^\star(k)\|_2^2}_{THD} + \delta \underbrace{\|\hat{f}_{sw}(k) - f_{sw}^\star\|_2^2}_{f_{sw}}
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Optimal Control Problem

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Infinite Horizon

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\begin{aligned}
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Impossible to Solve Online

Problem Solution

Optimal Control Problem

Infinite Horizon

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$$

Optimal Control Problem

Short Finite Horizon

$$
\begin{array}{ll}
\text{minimize} & \displaystyle\sum_{k=0}^{\boldsymbol{N-1}} \gamma^k l(x(k)) + \boldsymbol{\gamma^N V(x(k))} \\
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& x(0) = x_0 \\
& x(k) \in \mathcal{X},\, u(k) \in \{-1, 0, 1\}^3
\end{array}
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Very Short Horizon N

Optimal Control Problem

Short Finite Horizon

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& x(k) \in \mathcal{X}, u(k) \in \{-1, 0, 1\}^3
\end{aligned}
$$

Very Short Horizon N

Approximate $V(x(k))$ Offline

Bellman Equality

$$V^\star(x) = \min_u \left\{ l(x) + \gamma V^\star\left(Ax + Bu\right) \right\}$$

Bellman Equality

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Bellman Operator $\mathcal{T}$

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Contractive $\quad \lim_{M \to \infty} \mathcal{T}^M V = V^\star$

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Bellman Operator $\mathcal{T}$

Contractive $\lim_{M\to\infty} \mathcal{T}^M V = V^\star$

Monotone $V_1 \leq V_2 \implies \mathcal{T}V_1 \leq \mathcal{T}V_2$

Sufficient condition for underestimating $V^\star$

$$V \leq \mathcal{T}V \implies V \leq V^\star$$

Infinite Dimensional LP

$$\begin{array}{ll}\text{maximize} & \displaystyle\int_{\mathcal{X}} V(x)c(\mathrm{d}x)\\ \text{subject to} & V(x) \leq \mathcal{T}V(x) \quad \forall x\end{array}$$

Infinite Dimensional LP

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\begin{array}{ll}
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Iterated Bellman Inequality

Infinite Dimensional LP

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Iterated Bellman Inequality

Restrict to Quadratic Functions

$$V(x) = x^\top P x + 2q^\top x + r$$

Infinite Dimensional LP

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Iterated Bellman Inequality

Restrict to Quadratic Functions

$$V(x) = x^\top P x + 2q^\top x + r$$

$\longrightarrow$

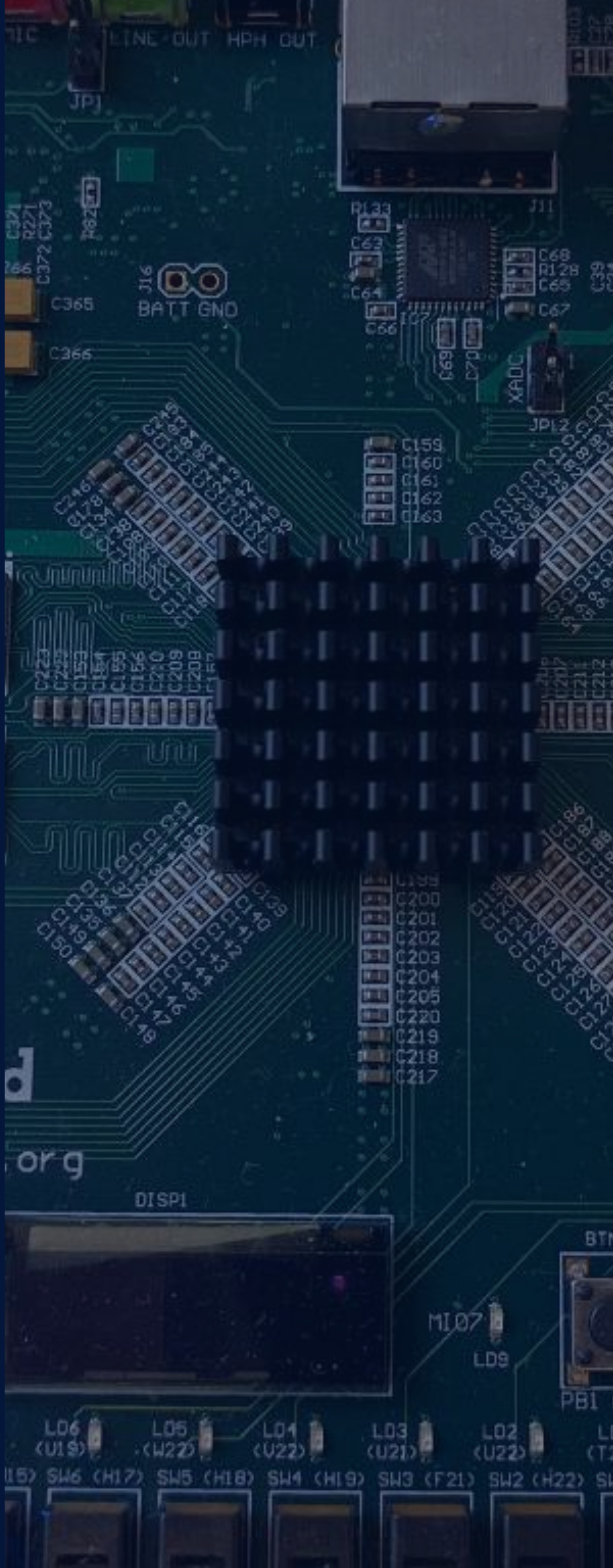
Tractable SDP (Offline)

FPGA Implementation

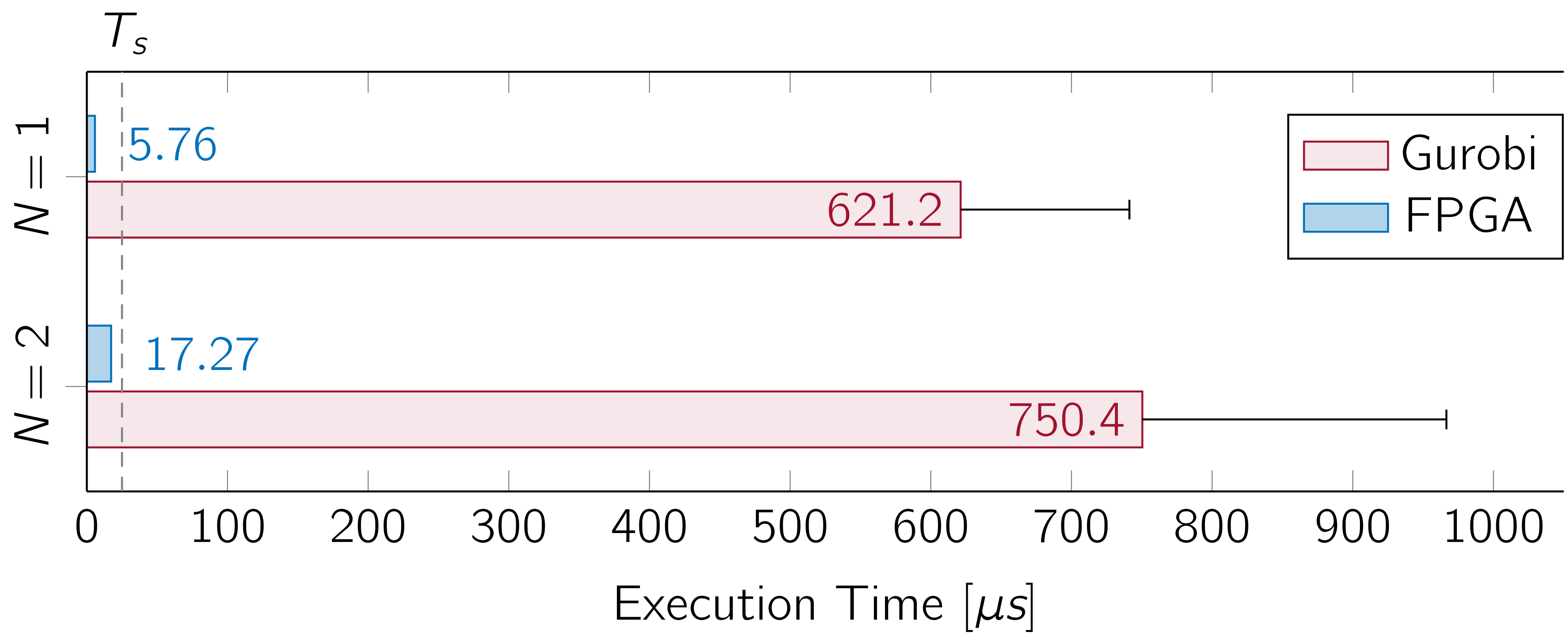
PROTOIP Toolbox

(Suardi et al.)

- Brute force enumeration
- Pipelined Evaluation of Switch Sequences
- Parallelized Matrix Computations



Timing Benchmarks



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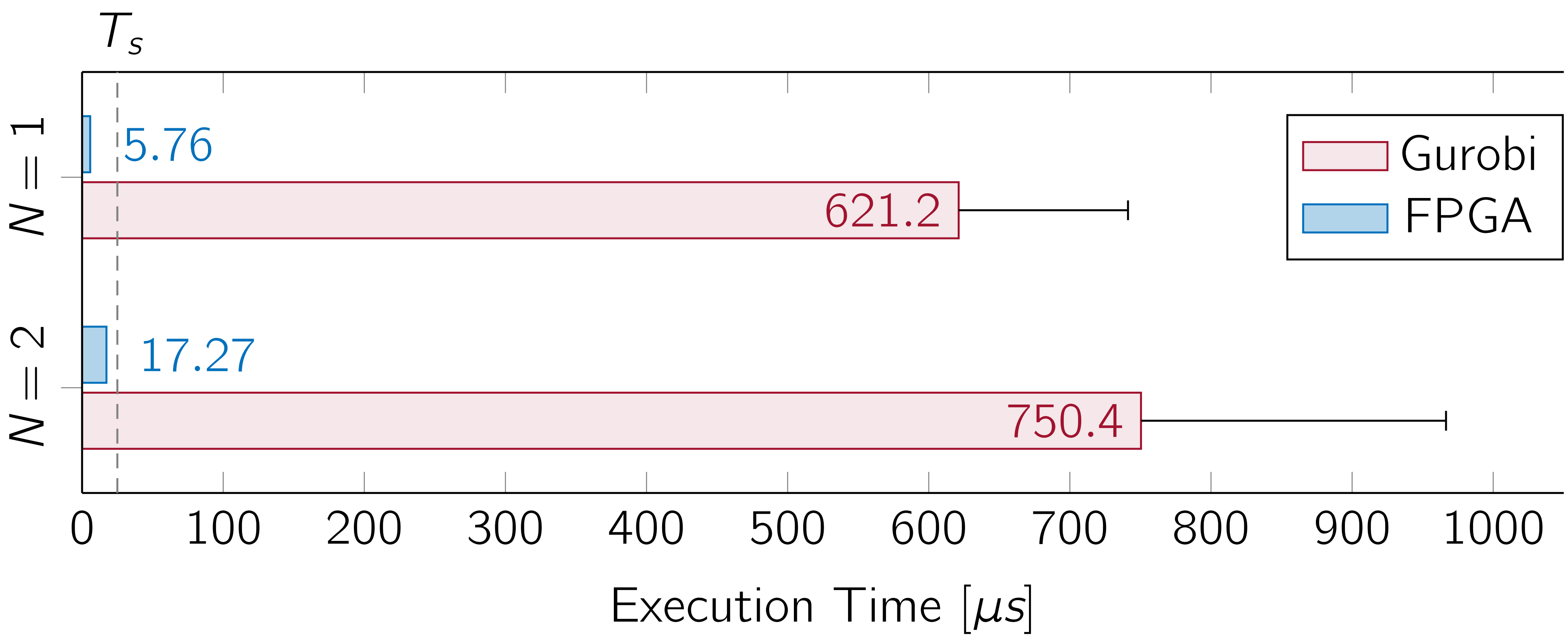
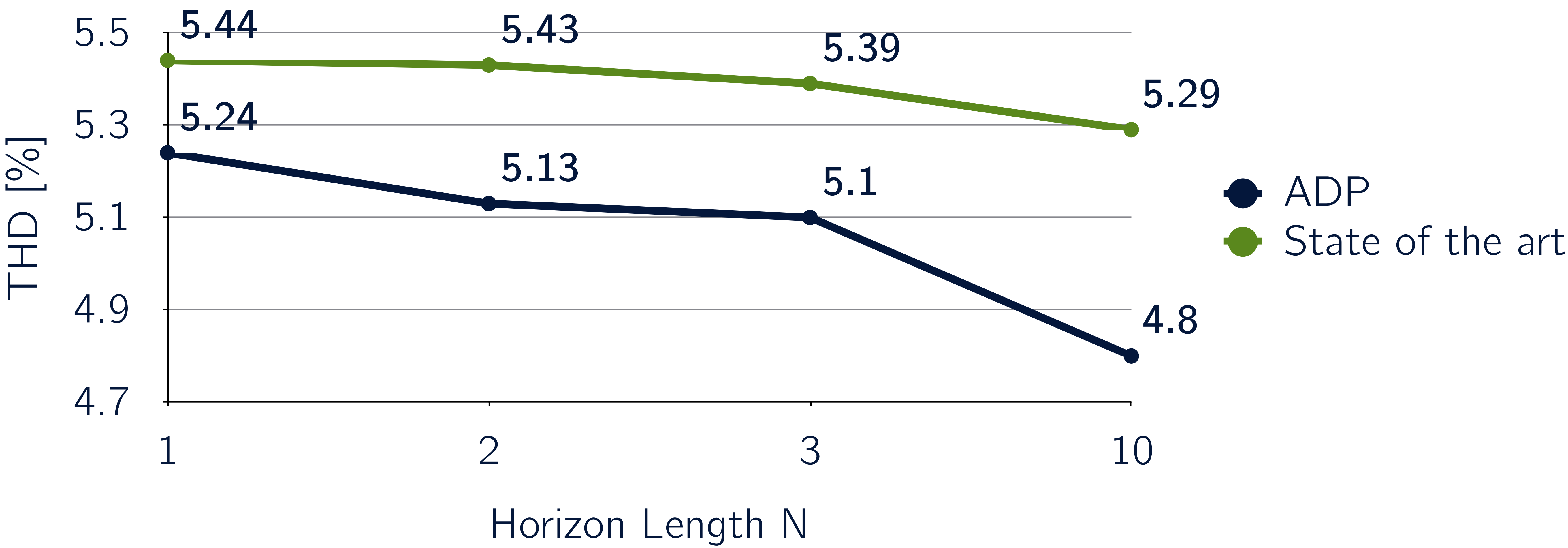


Computation Times under 25 μs!

Hardware in the Loop Tests

Steady-State Performance at $f_{sw} = 300$ Hz



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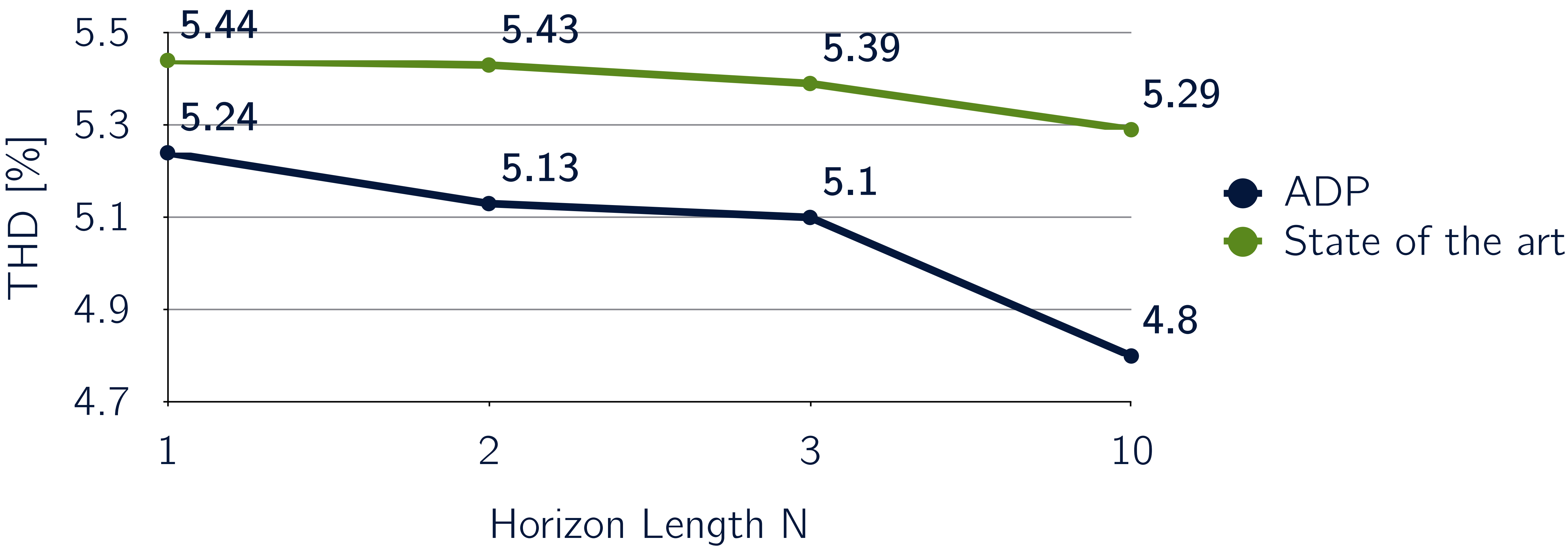
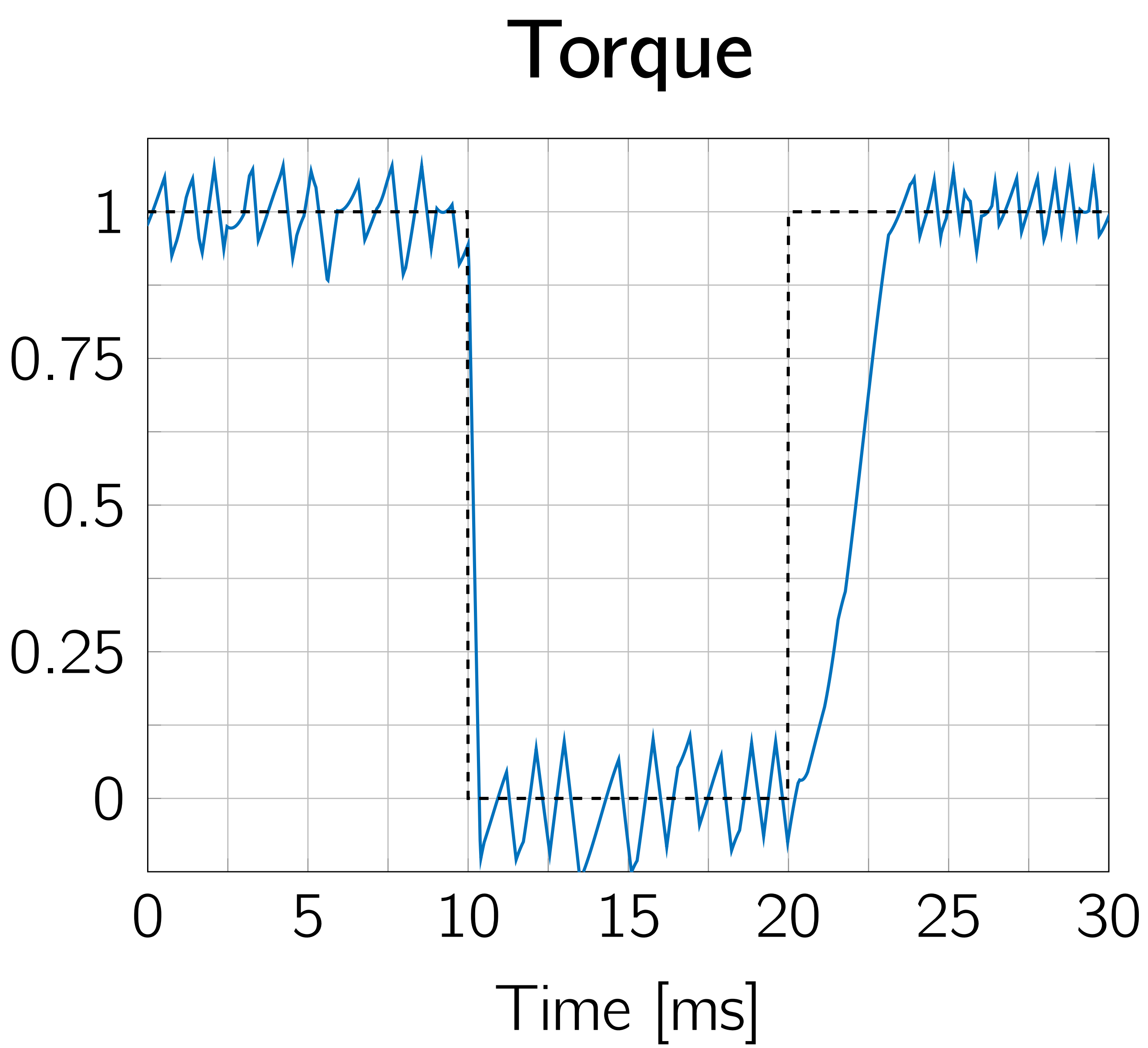
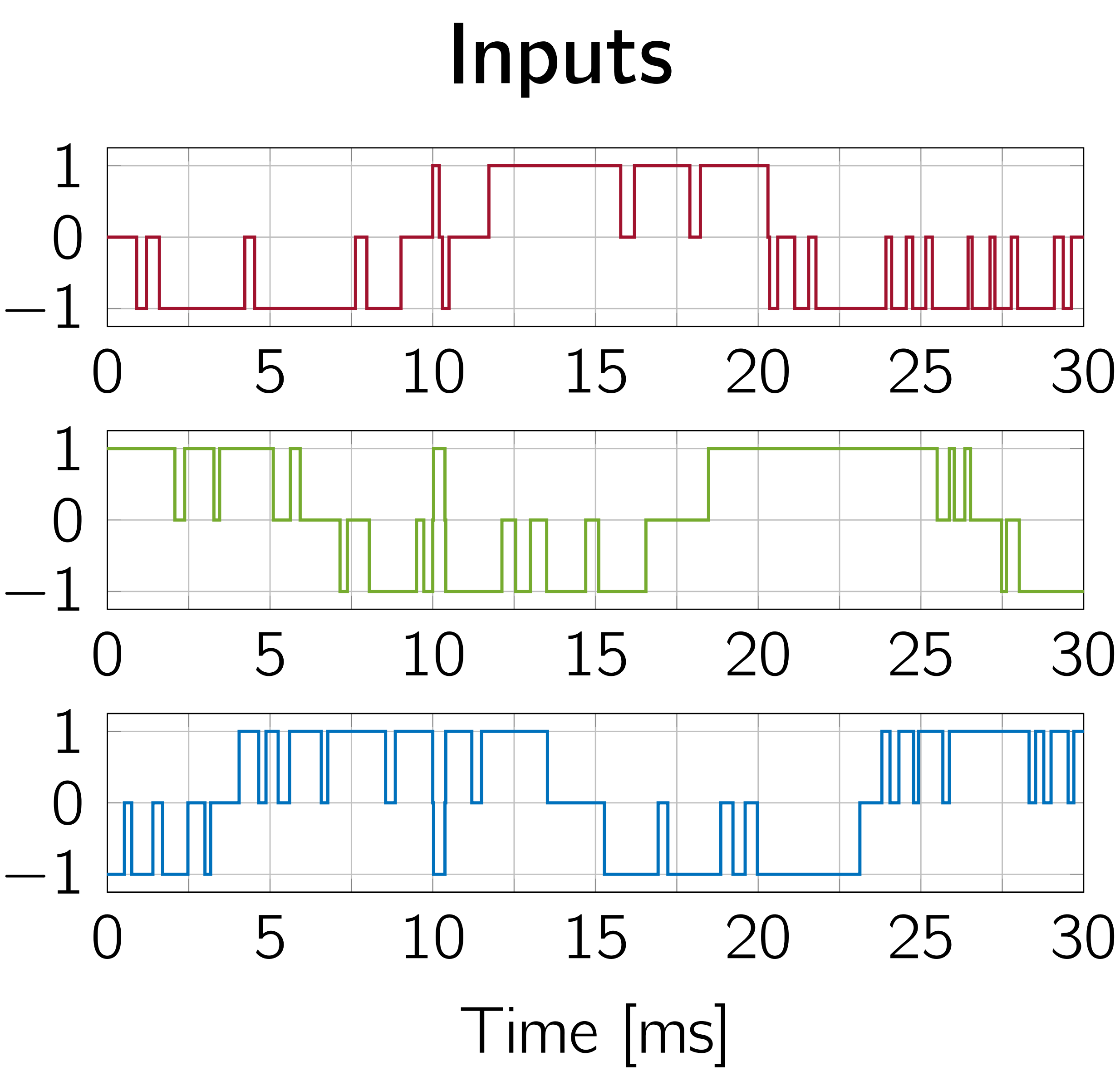
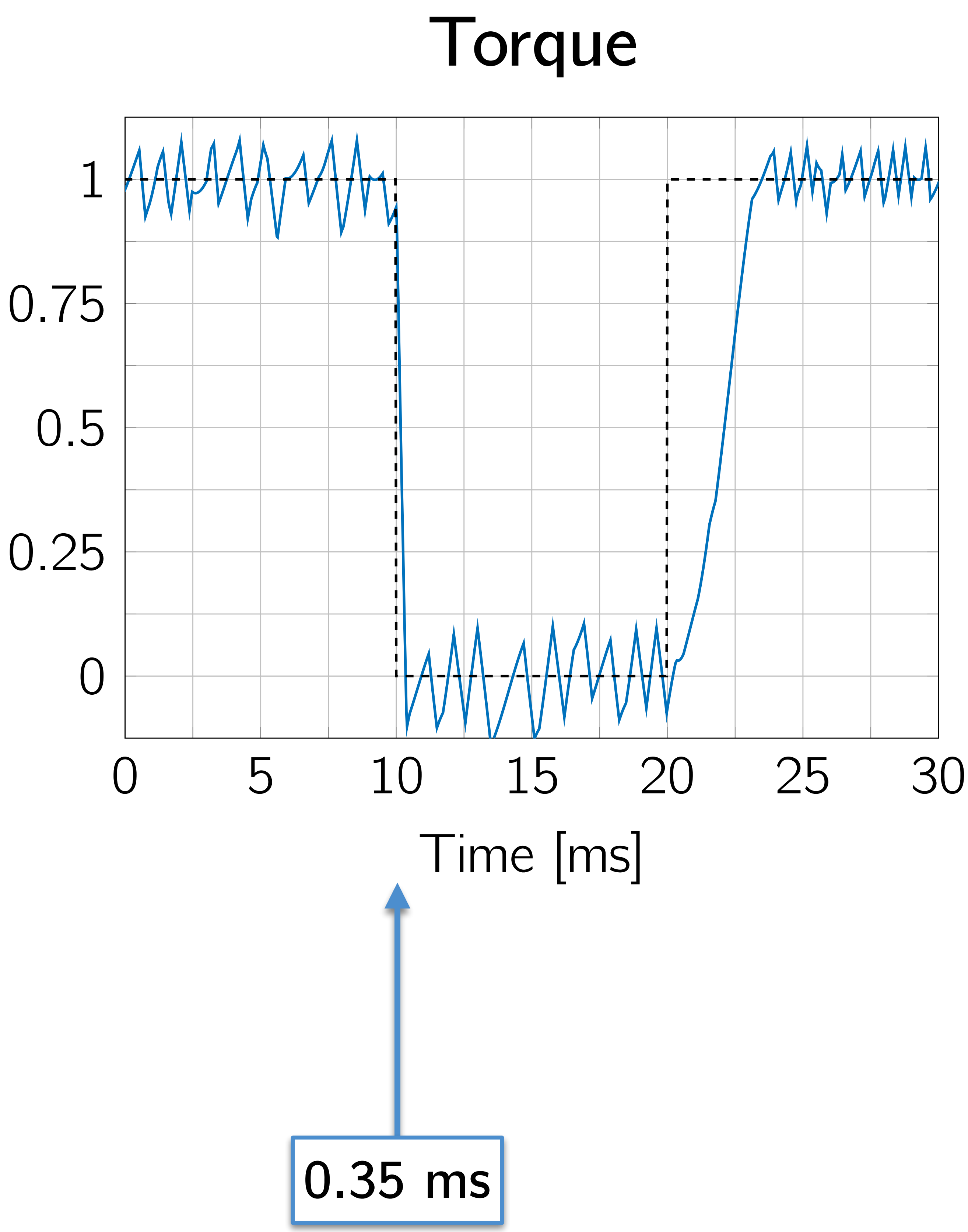
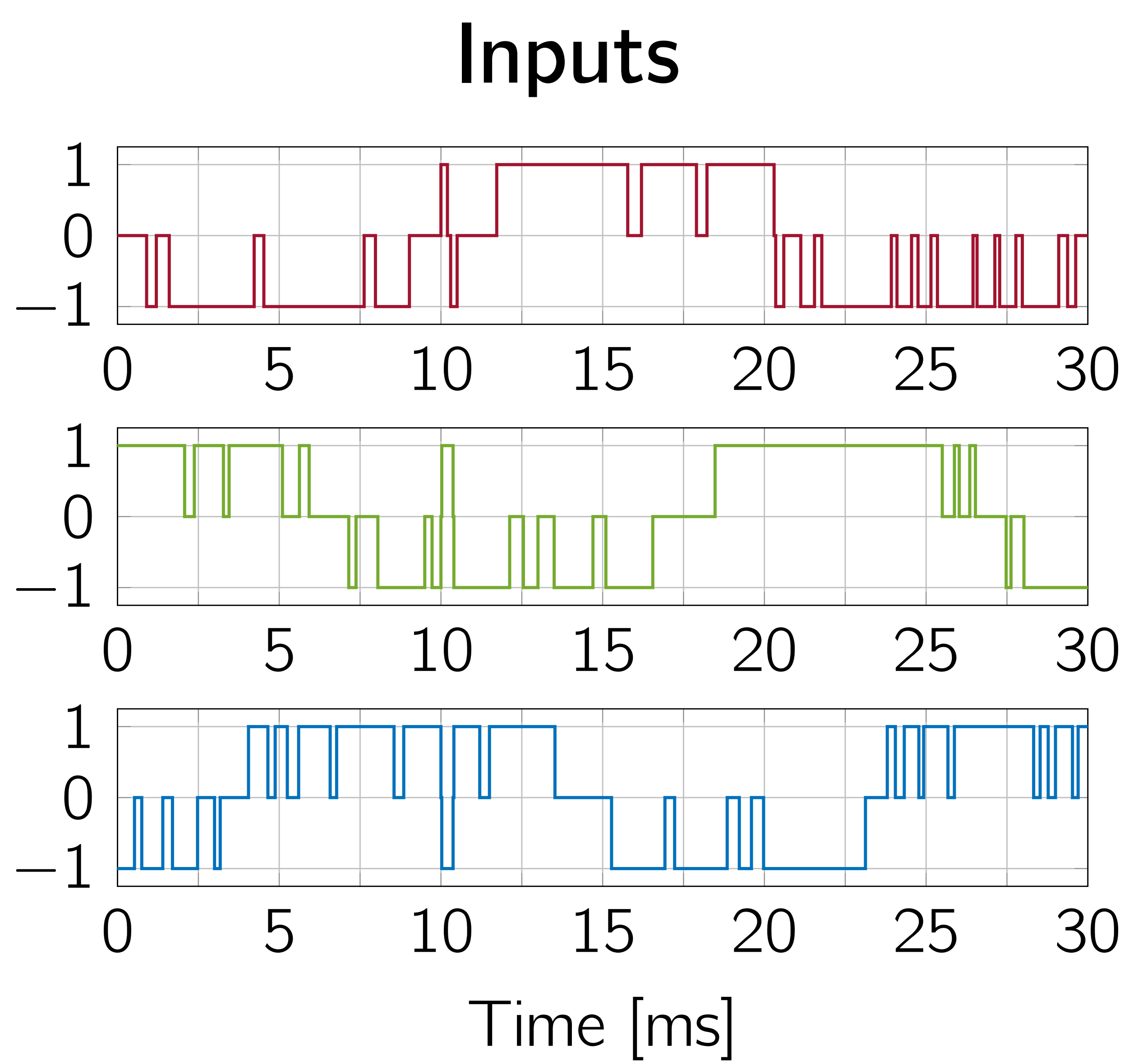


Better Performance with Short Horizons


Torque
1
0.75
0.5
0.25
0
0
5
10
15
20
25
30
Time [ms]

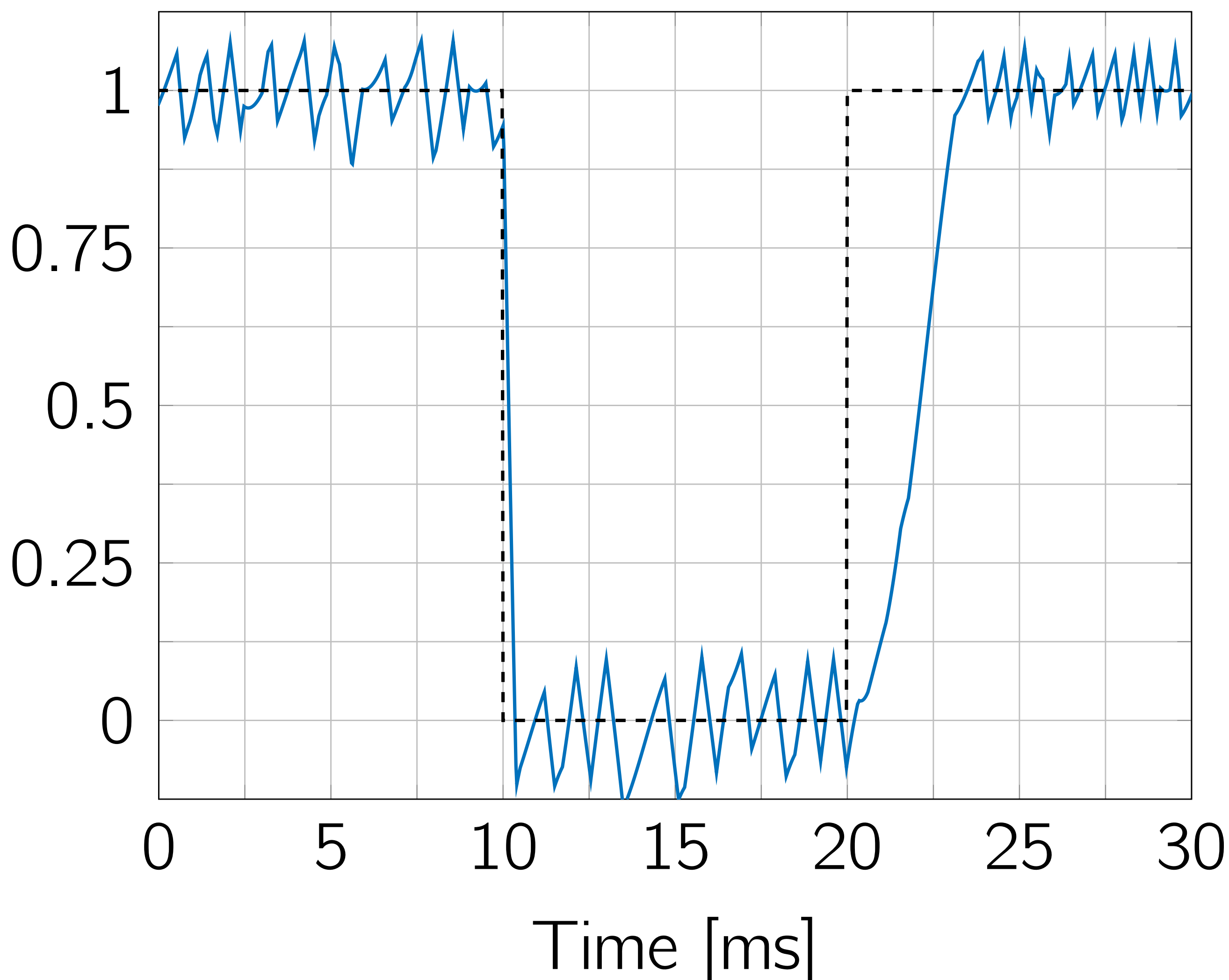

Inputs
1
0
−1
0
5
10
15
20
25
30
Time [ms]


Torque
1
0.75
0.5
0.25
0
0 5 10 15 20 25 30
Time [ms]
0.35 ms


Inputs
1
0
−1
0 5 10 15 20 25 30
Time [ms]

Transients

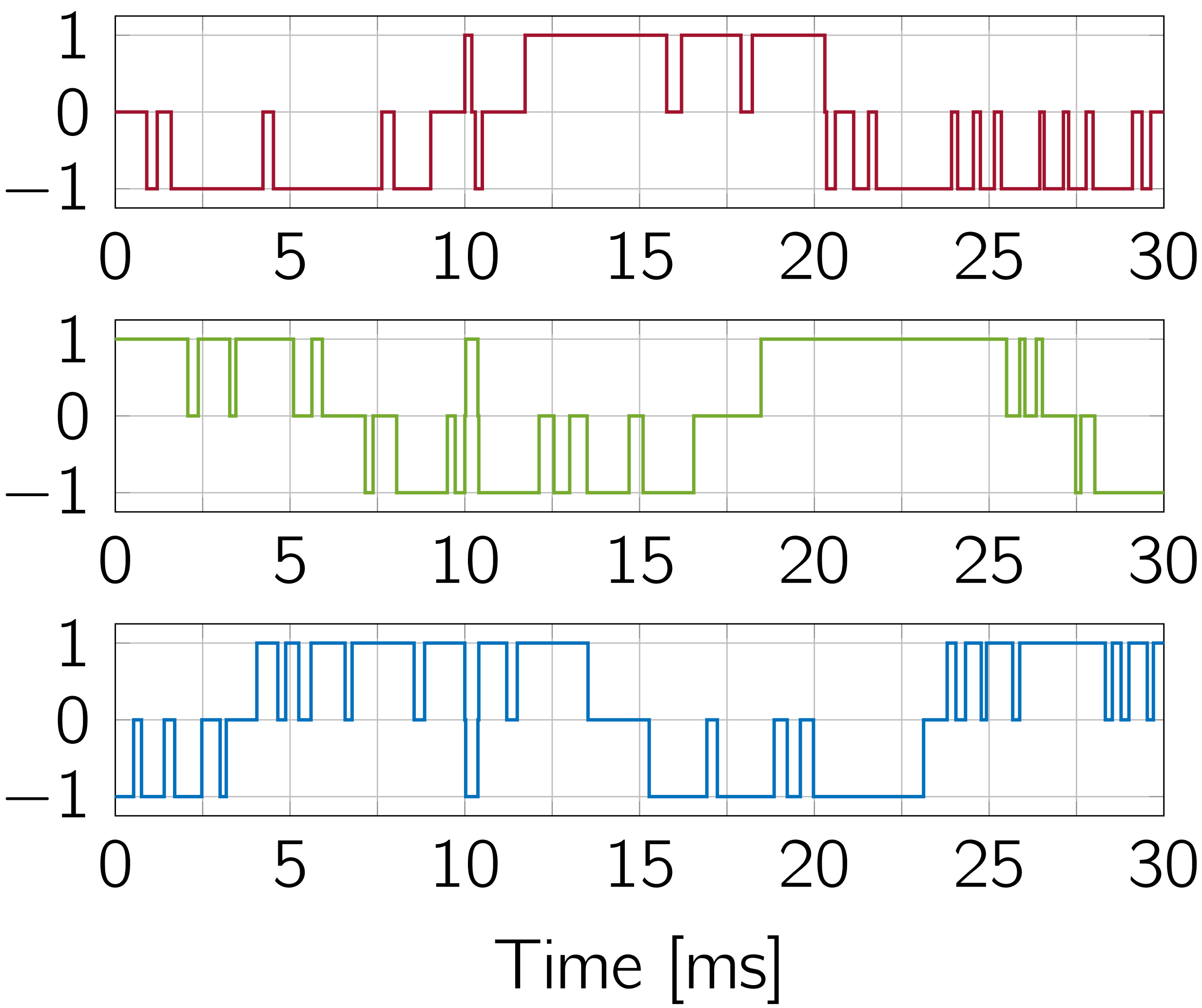
Torque



0.35 ms

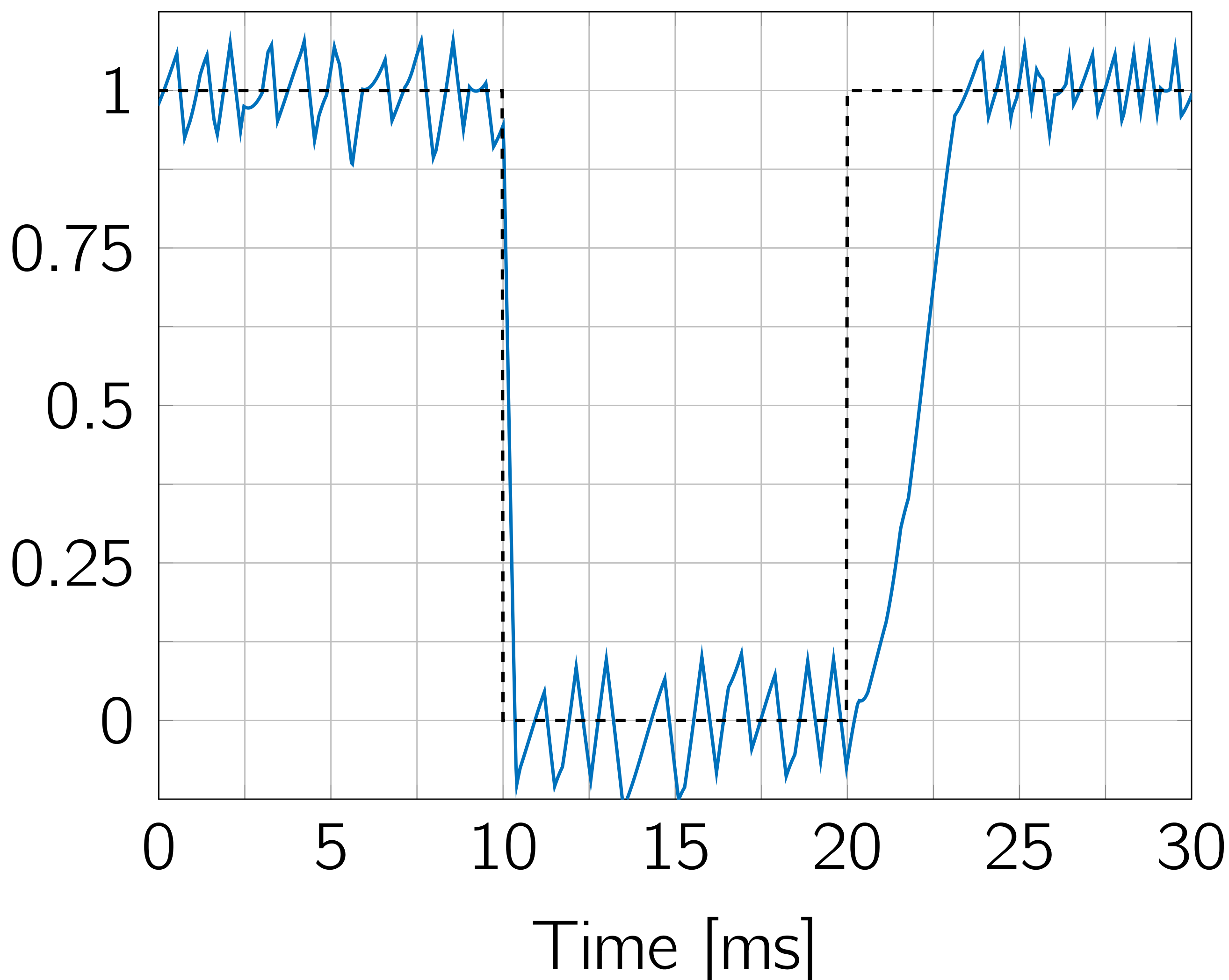
3.5 ms

Inputs



Transients

Torque



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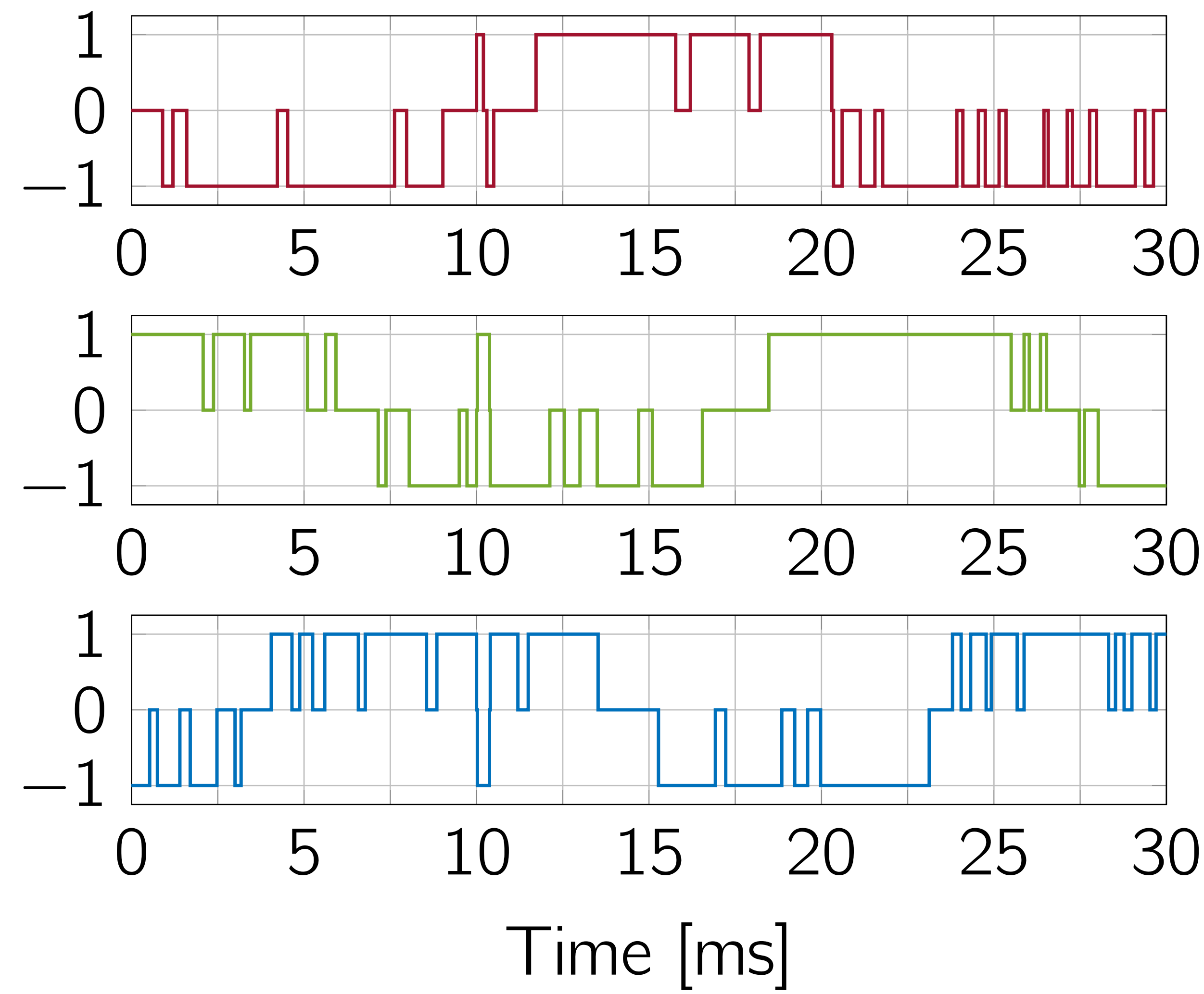
Inputs



Very Fast Transient Times!

Conclusions - New Approach to Direct MPC

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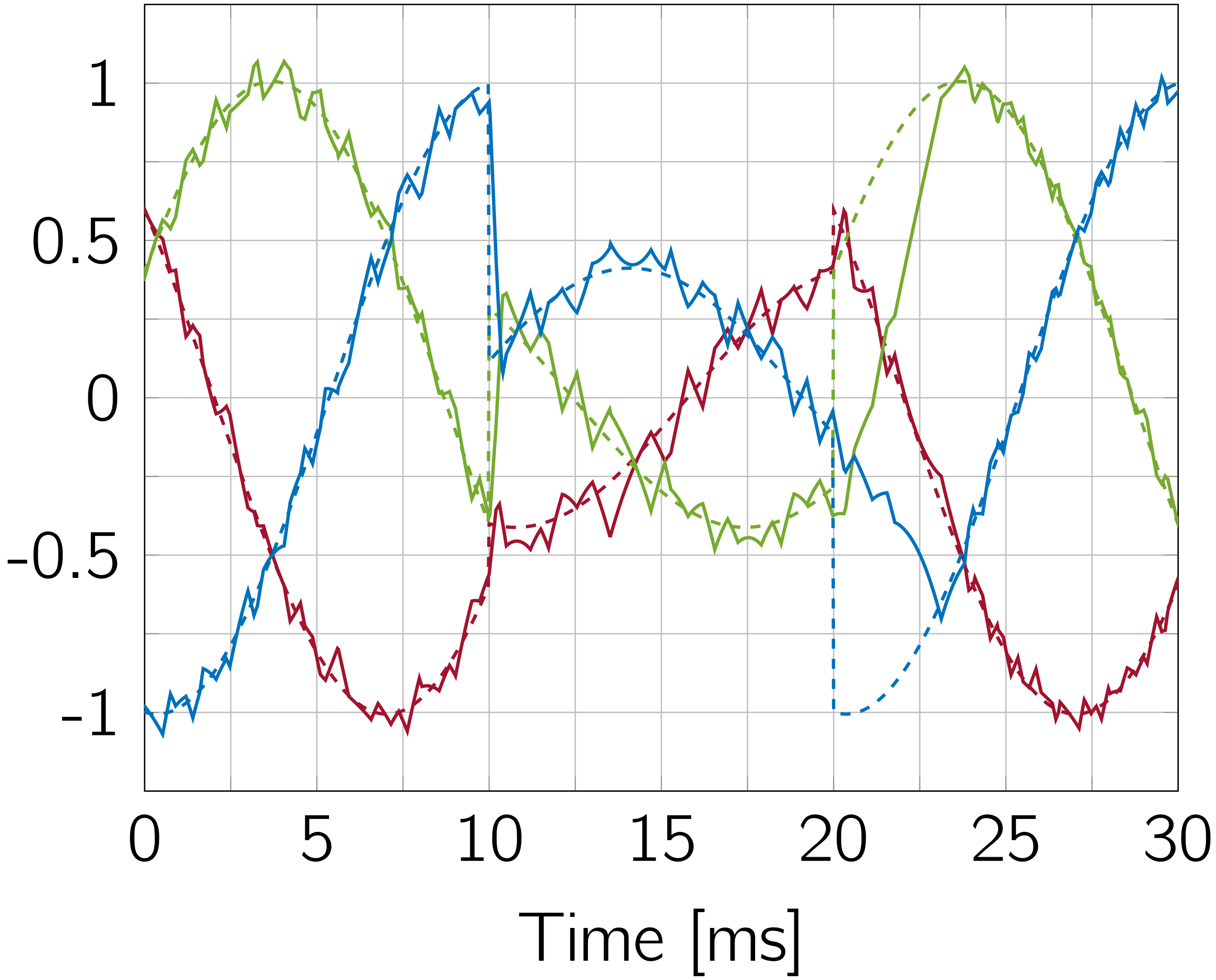
"High-Speed Finite Control Set Model Predictive Control for Power Electronics"
B. Stellato, T. Geyer, P. J. Goulart
IEEE Transactions on Power Electronics (In Press)

$$T = C(\Psi_r \times i_s)$$



Integer Program

$$
\begin{array}{ll}
\text{minimize} & \frac{1}{2}U^\top QU + f(x_0)^\top U \\
\text{subject to} & A_{ineq}U \le b_{ineq}(x_0) \\
 & U \in \{-1, 0, 1\}^{3N}
\end{array}
$$

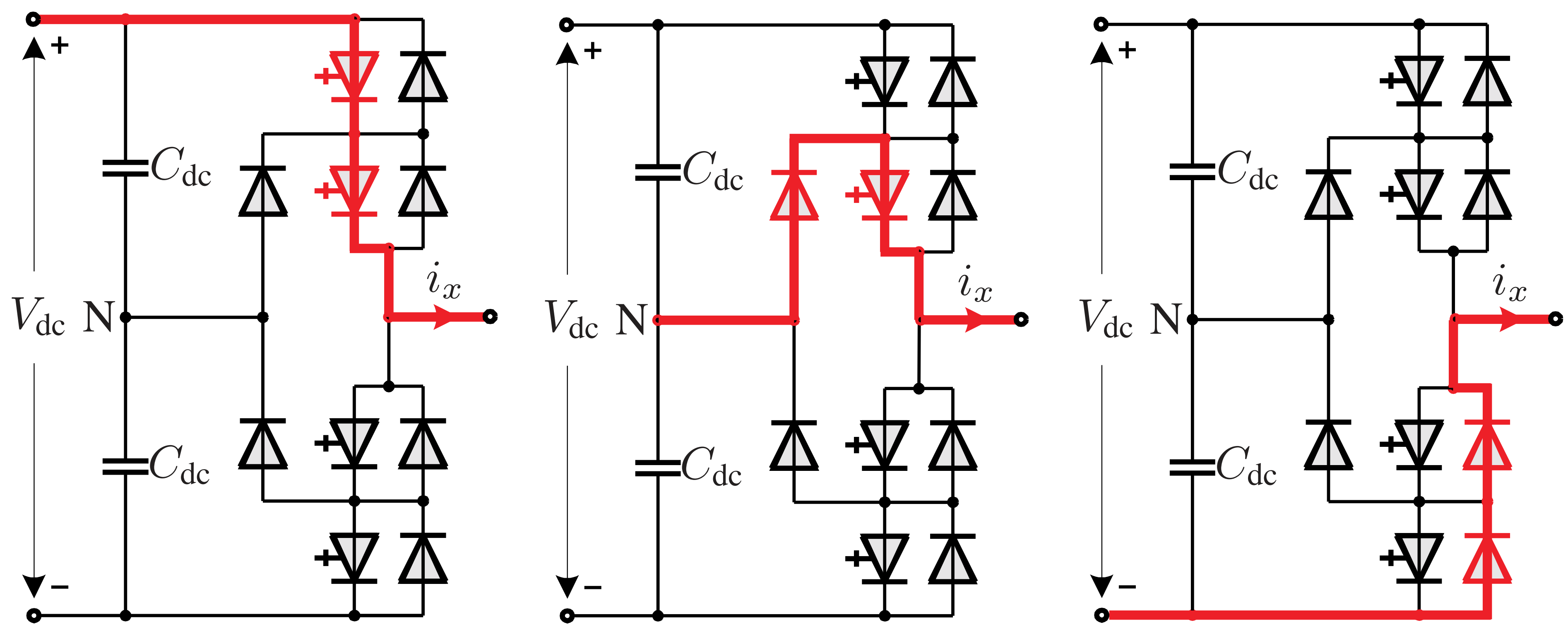
Backup - SDP Formulation

Quadratic Cost Function

$$\int_{\mathcal{X}} V_0(z) c(\mathrm{d}z) = \mathrm{Tr}(P_0 \Sigma_c) + 2 q_0^\top \mu_c + r_0$$

SDP Formulation

$$\begin{array}{ll}
\text{maximize} & \mathrm{Tr}\left(P_0 \Sigma_c\right) + 2 q_0^\top \mu_c + r_0 \\
\text{subject to} & \tilde{M}_i(m) \succeq 0, \quad \forall m \in \mathcal{M}, \quad i = 1, \dots, M \\
& V_0 = V_M \\
& P_i \in \mathbb{S}^{n_x},\ q_i \in \mathbb{R}^{n_x},\ r_i \in \mathbb{R},\ i = 0, \dots, M
\end{array}$$



V_{dc}
N
C_{dc}
C_{dc}
i_x
+
−